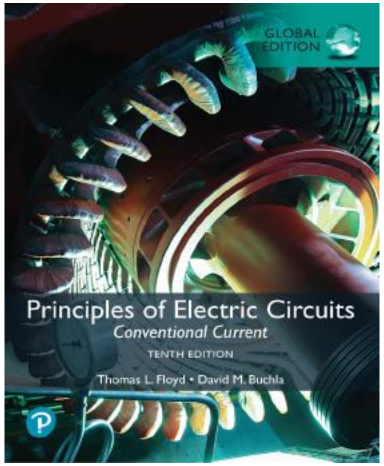
Principles of Electric Circuits: Conventional Current

Tenth Edition, Global Edition



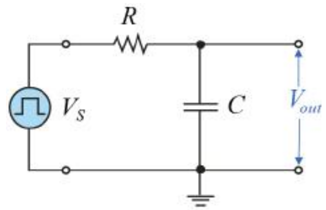
Chapter 20

Time Response of Reactive Circuits

Summary: The *RC* Integrator (1 of 5)

An *RC* integrator is a circuit that approximates the mathematical process of integration. Integration is a summing process, and a basic integrator can produce an output that is a running sum of the input under certain conditions.

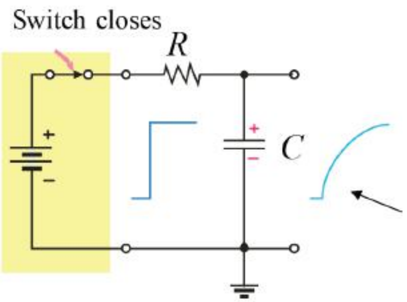
A basic *RC* integrator circuit is simply a capacitor in series with a resistor and the source. The output is taken across the capacitor.



Summary: The *RC* Integrator (2 of 5)

When a pulse generator is connected to the input of an *RC* integrator, the capacitor will charge and discharge in response to the pulses.

When the input pulse goes HIGH, the source acts as a battery in series with a switch.

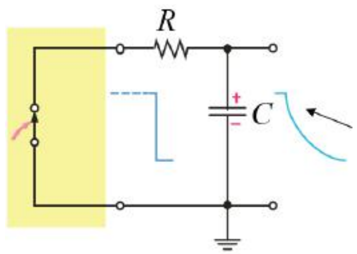


The output is an exponentially rising curve. Only the first part of this looks like true mathematical integration.

Summary: The *RC* Integrator (3 of 5)

When the pulse generator goes low, the internal impedance of the generator makes it look like a closed switch has replaced the battery.

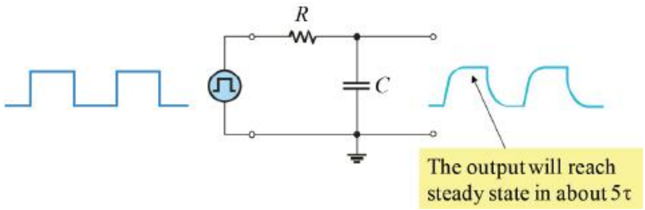
Battery is replaced with a switch that closes.



The output is an exponentially falling curve. Again, only the first part of this looks like true mathematical integration.

Summary: The *RC* Integrator (4 of 5)

Waveforms for the *RC* integrator depend on the time constant (τ) of the circuit. If the time constant is short compared to the period of the input pulses, the capacitor will fully charge and discharge. For an *RC* circuit, $\tau = RC$.



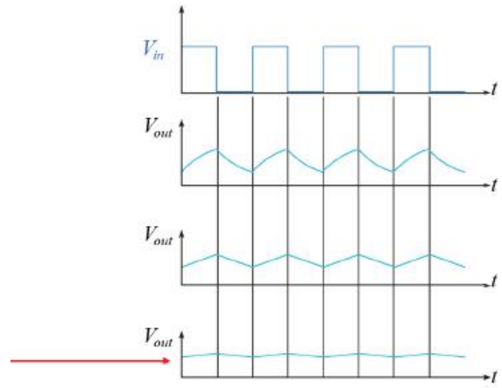
Example

What is the time constant if $R = 10\ \text{k}\Omega$ and $C = 0.022\ \mu\text{F}$?
220 μs

Summary: The *RC* Integrator (5 of 5)

If τ is increased, the waveforms approach the average dc level as in the last waveform. The output will appear triangular but with a smaller amplitude.

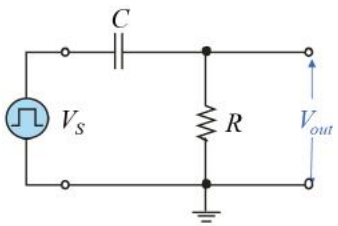
Alternatively, the input frequency can be increased (T shorter). The waveforms will again approach the average dc level.



Summary: The *RC* Differentiator (1 of 5)

An *RC* differentiator is a circuit that approximates the mathematical process of differentiation. Differentiation is a process that finds the rate of change, and a basic differentiator can produce an output that is the rate of change of the input under certain conditions.

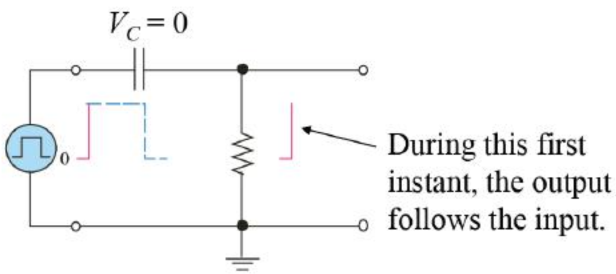
A basic *RC* differentiator circuit is simply a resistor in series with a capacitor and the source. The output is taken across the resistor.



Summary: The *RC* Differentiator (2 of 5)

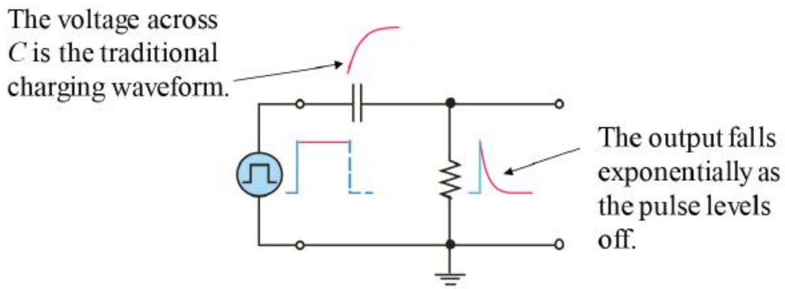
When a pulse generator is connected to the input of an *RC* differentiator, the capacitor appears as an instantaneous short to the rising edge and passes it to the resistor.

The capacitor looks like a short to the rising edge because voltage across *C* cannot change instantaneously.



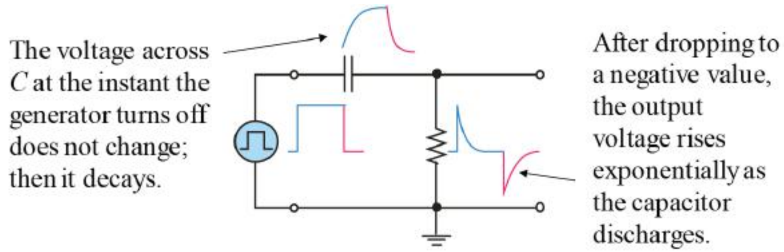
Summary: The *RC* Differentiator (3 of 5)

After the initial edge has passed, the capacitor charges and the output voltage decays.



Summary: The *RC* Differentiator (4 of 5)

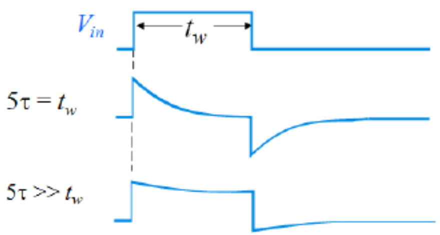
The falling edge is a rapid change, so it is passed to the output because the capacitor voltage cannot change instantaneously. The type of response shown happens when τ is much less than the pulse width ($\tau << t_w$).



Summary: The *RC* Differentiator (5 of 5)

The output shape is dependent on the ratio of τ to t_w.

When $5\tau = t_w$, the pulse has just returned to the baseline when it repeats.

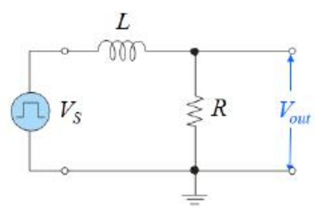


If τ is long compared to the pulse width, the output does have time to return to the original baseline before the pulse ends. The resulting output looks like a pulse with “droop”.

Summary: The *RL* Integrator (1 of 5)

Like the *RC* integrator, an *RL* integrator is a circuit that approximates the mathematical process of integration. Under equivalent conditions, the waveforms look like the *RC* integrator. For an *RL* circuit, $\tau = L/R$.

A basic *RL* integrator circuit is a resistor in series with an inductor and the source. The output is taken across the resistor.



Example

What is the time constant if $R = 220\ \Omega$ and $L = 22$ mH?
100 µs

The next slide illustrates how to measure t using Multisim.

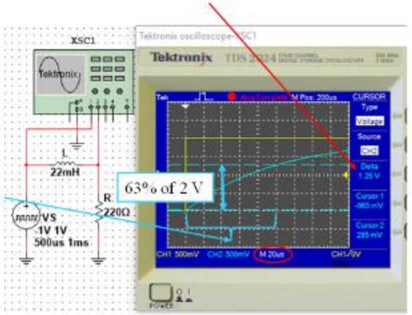
Summary: The *RL* Integrator (2 of 5)

CH-1 (yellow trace) is across the source; CH-2 (cyan trace) is across the inductor. The scope is equipped with cursors to simplify measurements. Set voltage cursors to 0 and 63% of the input amplitude (1.26 V in this case).

Count the number of horizontal divisions from the start of the pulse until the output crosses the upper cursor. (5.0 divisions)

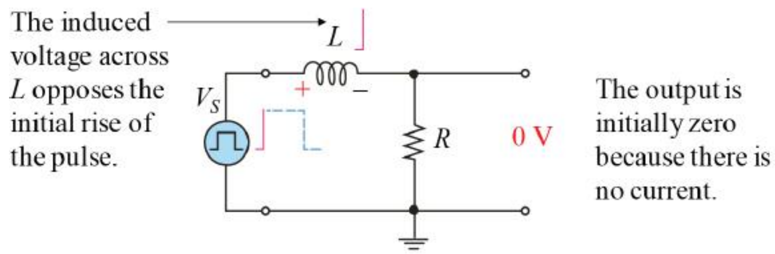
Multiply the number of divisions by the time/division (20 μs)

The result is τ. (100 μs)



Summary: The *RL* Integrator (3 of 5)

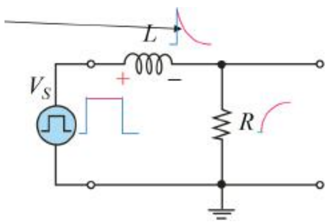
When the pulse generator output rises, a voltage immediately appears across the inductor in accordance with Lenz's law. The instantaneous current is zero, so the resistor voltage is initially zero.



Summary: The *RL* Integrator (4 of 5)

At the top of the input pulse, the inductor voltage decays exponentially and current rises. As a result, the voltage across the resistor rises exponentially.

The induced voltage across *L* decays.

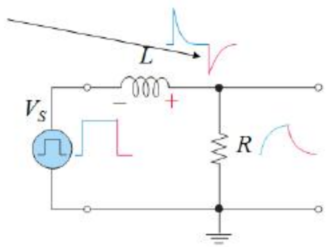


The output voltage rises as current builds in the circuit.

Summary: The *RL* Integrator (5 of 5)

When the pulse falls, a reverse voltage is induced across *L* opposing the change. The inductor voltage initially is a negative voltage that is equal and opposite to the generator; then it exponentially rises.

The induced voltage across *L* initially opposes the change in the source voltage.



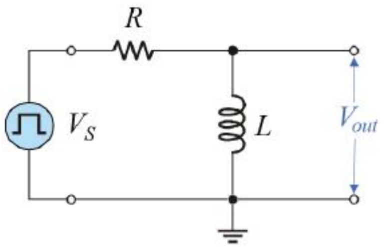
The output voltage decays as the magnetic field around *L* collapses.

Note that these waveforms were the same in the RC integrator.

Summary: The *RL* Differentiator (1 of 4)

An *RL* differentiator is also a circuit that approximates the mathematical process of differentiation. It can produce an output that is the rate of change of the input under certain conditions.

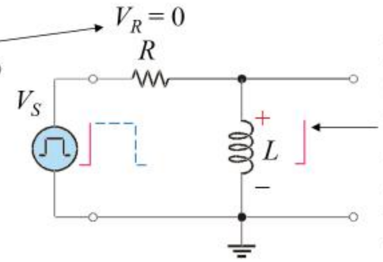
A basic *RL* differentiator circuit is an inductor in series with a resistor and the source. The output is taken across the inductor.



Summary: The *RL* Differentiator (2 of 4)

When a pulse generator is connected to the input of an *RL* differentiator, the inductor has a voltage induced across it that opposes the source; initially, no current is in the circuit.

Current is initially zero, so $V_R = 0$.

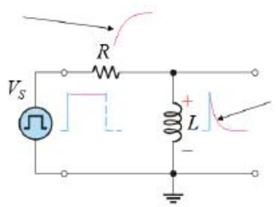


During this first instant, the inductor develops a voltage equal and opposite to the source voltage.

Summary: The *RL* Differentiator (3 of 4)

After the initial edge has passed, current builds in the circuit. Eventually, the current reaches a steady state value given by Ohm’s law.

The voltage across *R* rises as current increases.

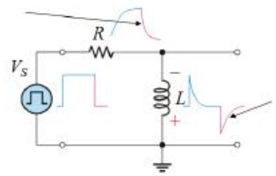


The output falls exponentially as the pulse levels off.

Summary: The *RL* Differentiator (4 of 4)

Next, the falling edge of the pulse causes a (negative) voltage to be induced across the inductor that opposes the change. The current decays as the magnetic field collapses.

The voltage across *R* decays as current decreases.



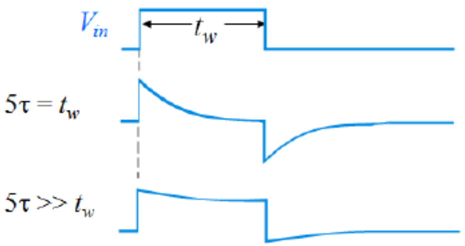
The output drops initially and then rises exponentially.

Summary: The *RL* Differentiator

As in the case of the *RC* differentiator, the output shape is dependent on the ratio of τ to t_w.

When $5\tau = t_w$, the pulse has just returned to the baseline when it repeats.

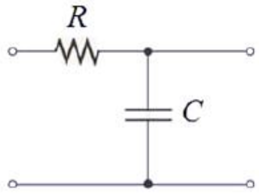
If τ is long compared to the pulse width, the output looks like a pulse with "droop".



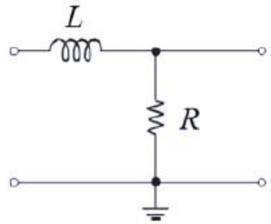
Summary: The integrator as a low-pass filter

Recall that basic filter circuits can be constructed from the same passive components used in integrator and differentiator circuits. A low-pass filter has the same arrangement of components as

an *RC* integrator:



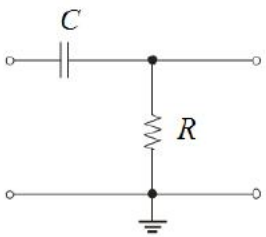
…or an *RL* integrator:



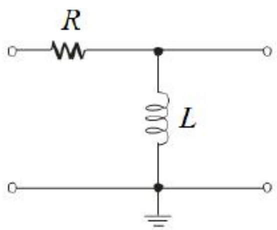
Summary: The differentiator as a high-pass filter

Likewise, a high-pass filter can be constructed from the same basic components as those used for differentiators. A high-pass filter has the same arrangement of components as

an *RC* differentiator:



…or an *RL* differentiator:



Summary: Rise time, fall time, and frequency

As you might expect, there is a close relationship between the rise (or fall) time in a pulse circuit and the frequency response of basic filters. A very useful relationship can be derived from rise (or fall) times as

$$f_h = \frac{0.35}{t_r}$$

where f_h is the highest frequency component in a pulse and t_r is the rise (or fall) time of the input pulse.

Example

What is the highest frequency component in a pulse that rises in 3.5 ns? 100 MHz

Key Terms

DC component	The average value of a pulse waveform.
Differentiator	A circuit producing an output that approaches the mathematical derivative of the input.
Integrator	A circuit producing an output that approaches the mathematical integral of the input.
Steady state	The equilibrium condition of a circuit that occurs after an initial transient time.
Transient time	An interval equal to approximately five time constants.

Quiz (1 of 11)

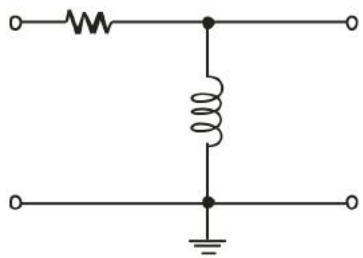
1. The circuit shown is

   a. an integrator.

   b. a high-pass filter

   c. both of the above

   d. none of the above



Quiz (2 of 11)

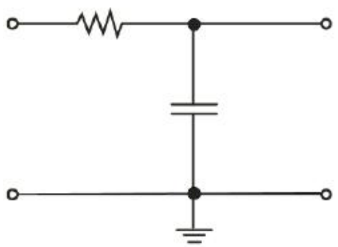
2. The circuit shown is

   a. an integrator.

   b. a low-pass filter

   c. both of the above

   d. none of the above



Quiz (3 of 11)

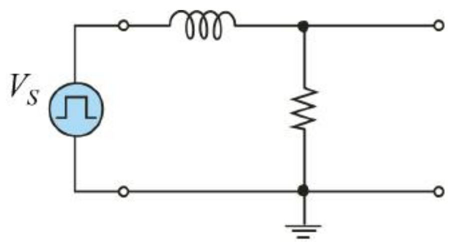
3. Initially, when the pulse from the generator rises, the voltage across *R* will be

   a. equal to the inductor voltage

   b. one-half of the inductor voltage

   c. equal to V_S

   d. zero



Quiz (4 of 11)

4. After an *RL* integrator has reached steady state from an input pulse, the output voltage will be equal to

   a. 1/2 V_S

   b. 0.63 V_S

   c. V_S

   d. zero

Quiz (5 of 11)

5. The time constant for an *RL* integrator is given by the formula

   a. $\tau = L/R$

   b. $\tau = 0.35RL$

   c. $\tau = R/L$

   d. $\tau = LR$

Quiz (6 of 11)

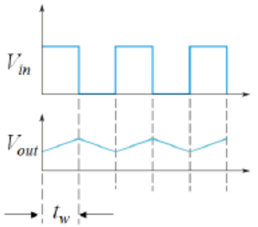
6. The input and output waveforms for an integrator are shown. From the waveforms, you can conclude that

   a. $\tau = t_w$

   b. $\tau >> t_w$

   c. $\tau << t_w$

   d. none of the above



Quiz (7 of 11)

7. If a 20 kΩ resistor is in series with a 0.1 μF capacitor, the time constant is

   a. 200 μs

   b. 0.5 ms

   c. 1.0 ms

   d. none of the above

Quiz (8 of 11)

8. After a single input transition from 0 to 10 V, the output of a differentiator will be back to 0 V in
   a. less than one time constant
   b. one time constant
   c. approximately five time constants
   d. never

Quiz (9 of 11)

9. The circuit for a low-pass filter is the same as the circuit for
   a. an integrator
   b. a differentiator
   c. both of the above
   d. none of the above

Quiz (10 of 11)

10. The highest frequency component in a pulse that rises in 2.5 ns is
   a. 140 MHz
   b. 100 MHz
   c. 71 MHz
   d. 35 MHz

Quiz (11 of 11)

Answers:

1. b
2. c
3. d
4. c
5. a
6. b
7. d
8. c
9. a
10. a