AI-C 205 Probability and Statistics

Homework 4
(Deadline: 27^{th} Oct, 2026)

1. My friend claims that she has a coin that has a probability 0.7 of coming up heads. To test her claim, I propose the following procedure: I will toss her coin ten times and reject her claim if the number of heads is less than five or greater than nine. How good is my proposed procedure?

2. Show that for large N and small π, the binomial distribution (N, π) is approximately Poisson with parameter $\lambda = N\pi$.

3. Derive the mean and variance of the uniform distribution on [a, b].

4. Let us say we have three machines in our factory and they respectively produce 50, 30, and 20 percent of our output. The machines are not equally good, and their probabilities of producing a defective item respectively are 3, 2, and 1 percent.

   (a) What is the overall probability of a defective product?

   (b) Let us say that we take a random item from our production and see that it is defective; what is the probability that it was produced by machine 2?