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FACULTY OF ENGINEERING

EEE 222 ELECTRICAL CIRCUIT LABORATORY II

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EXPERIMENT III

INTRODUCTION TO

AC POWER ANALYSIS

1. Objectives

The objective of this experiment is to analyze circuits energized by sinusoidal sources of voltage or current. Particularly, the concepts of phasors, complex representation of circuit elements, and power calculations.

2. Introduction

A sinusoidal voltage (or current) source produces a voltage (or current) that varies sinusoidally with time. A sinusoidal voltage source can be described mathematically as:

$$v(t) = V_m \cos(\omega t + \emptyset)$$

where V_m is the peak amplitude, ω is the angular frequency (in rad/sec), and $\emptyset$ is the phase angle.

When a sinusoidal source energizes a circuit, all the signals of interest in the circuit have the same frequency. Therefore, it is sufficient to represent those signals in terms of their amplitude and phase angle. This leads to the phasor representation of sinusoidal signals:

$$\boldsymbol{V} = V_m e^{j\emptyset} = V_m \angle \emptyset = V_m cos\emptyset + jV_m sin\,\emptyset = \mathcal{P}[V_m \cos(\omega t + \emptyset)]$$

Where $\mathcal{P}$ is the phasor transform (from the time domain to the frequency domain), whose inverse (from the frequency domain to the time domain) yields:

$$\mathcal{P}^{-1}\{V_m e^{j\emptyset}\} = \mathcal{R}\{V_m e^{j\emptyset} e^{j\omega t}\} = v(t)$$

In a resistor, the sinusoidal voltage and current are in phase. At the terminals of an inductor the voltage leads the current by 90°, and at the terminals of a capacitor the current leads the voltage by 90°. The relationship between phasor current and phasor voltage for resistors, inductors, and capacitors is

$$\boldsymbol{V} = Z\boldsymbol{I}$$

Where Z is the ***impedance*** (in Ω) of the element. The reciprocal of impedance is ***admittance*** (Y, in mhos ℧), which relates the phasor voltage and phasor current of an element:

$$\boldsymbol{V} = \boldsymbol{I}/Y$$

In general, impedance and admittance are complex numbers. The imaginary part of impedance is called ***reactance***, and the imaginary part of admittance ***susceptance***. Table below summarizes the values of impedance, reactance, admittance and susceptance for resistors, inductors, and capacitors.

Element	Impedance	Reactance	Admittance	Susceptance
Resistor	R (Resistance)	-	G (Conductance)	-
Capacitor	$1/j\omega C$	$-1/\omega C$	$j\omega C$	ωC
Inductor	$j\omega L$	ωL	$1/j\omega L$	$-1/\omega L$

All the techniques used in DC circuit analysis may be used to analyze a frequency domain circuit using phasors. Phasor diagrams are commonly used to visualize the relationship among the different signals in the frequency domain.

When dealing with sinusoidal sources and the signals generated by them in a circuit it is often desirable to determine the power delivered to and dissipated by the circuit elements. The importance of discussing such calculations roots in that the voltage and current phasors may have different phase angles, hence, determining power is not limited to the product of the magnitudes of voltages and currents. In fact, there are several types of power that arise in a circuit operating in the sinusoidal steady-state.

Instantaneous power

The instantaneous power (in watts W) delivered to a load is a function of time defined as the product of the instantaneous terminal voltage and current:

$$p(t) = v(t)i(t)$$

The instantaneous power can be positive or negative, depending on whether the power is being dissipated by the load or is being delivered by the load. The frequency of the instantaneous power is twice the frequency of the voltage (or current).

Average power

For many purposes, an average value of the power over time is more useful than instantaneous power. It is the power that is converted from electrical to another form of energy (e.g. heat), for this reason it is often referred to as real power. Its units are also watts. The average value of the instantaneous power over one period is expressed as

$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)$$

Reactive power

Reactive power is the electric power exchanged between an inductor (or a capacitor) and the source that drives it. Reactive power is never converted to nonelectric power. Its units are var (volt amp reactive, or VAR), and is expressed as

$$Q = \frac{1}{2}V_m I_m \sin(\theta_v - \theta_i) = V_{rms} I_{rms} \sin(\theta_v - \theta_i)$$

Complex power

The complex sum of the real and reactive powers is the complex power. Its units are volt-amp (VA). Complex power is expressed as

$$S = P + jQ = \frac{1}{2}\boldsymbol{V}\boldsymbol{I}^* = \boldsymbol{V_{rms}}\boldsymbol{I_{rms}^*} = I_{rms}^2 Z = \frac{V_{rms}^2}{Z^*}$$

Apparent power

The magnitude (in VA) of the complex power is referred to as apparent power:

$$S = \sqrt{P^2 + Q^2}$$

Associated with reactive loads is the parameter known as ***power factor***, which is the cosine of the phase angle between the voltage and the current:

$$pf = \cos(\theta_v - \theta_i)$$

Similarly, the ***reactive factor*** is the sine of the phase angle between the voltage and the current:

$$rf = \sin(\theta_v - \theta_i)$$

Generally, it is preferred to have circuits (systems) with a power factor close to unity such that the real power is maximized. In circuits operating in the sinusoidal steady-state the ***maximum power transfer*** occurs when the load impedance is the complex conjugate of the Thevenin impedance as viewed from the terminals of the load impedance.

3. Preliminary Work

3.1. Consider the circuit in the Figure 3.1 and the component values of Table 3.1. Consider V_{in} as the reference phasor, i.e. assume V_{in} has magnitude 1 and phase zero at all frequencies disregarding V_s. Complete the entries of Table 3.2 corresponding to the theoretical values. Include the calculations in a separate sheet.

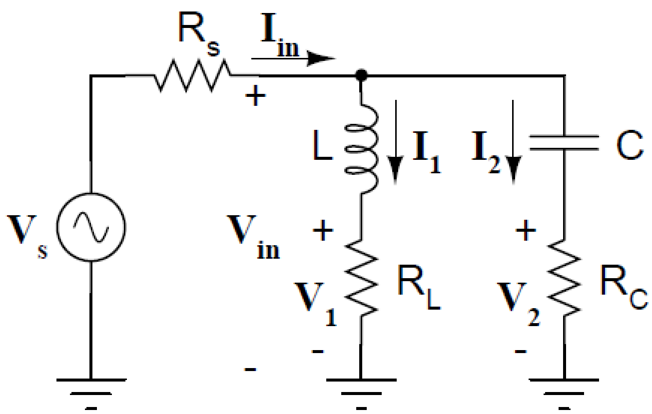

Figure 3.1. Phasor test circuit.

Parameter	Value	Parameter	Value
R_S	47 ohm	$V_{S\,(PP)}$	10 V
R_L	100 ohm	L	1 mH
R_C	100 ohm	C	2.2 μF

Table 3.1. Phasor test circuit component values.

Frequency (kHz)	Phasor	Theoretical		Experimental	
		Mag. (V)	Phase (°)	Mag. (V)	Phase (°)
1.8	V_{in}	1	0		0
	V_1				
	V_2				
18	V_{in}	1	0		0
	V_1				
	V_2				

Table 3.2. Phasor table for test circuit.

3.2. Consider the circuit of Figure 3.2 when the capacitor is connected as indicated by dashed lines. Derive an expression for the value of the input frequency in Hertz as a function of L, C and R such that the impedance of the load is real (i.e. the imaginary part is zero).

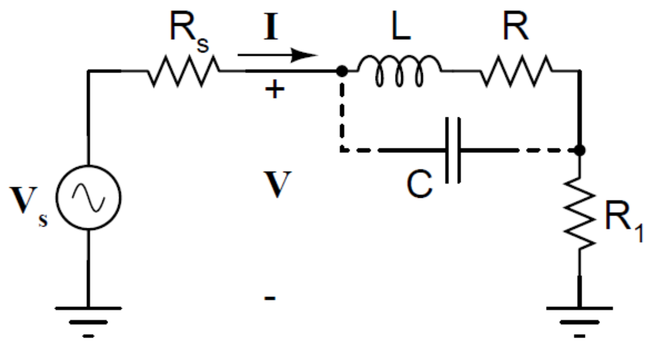

Figure 3.2. Power calculations test circuit.

3.3. Consider the circuit of Figure 3.3 and the component values of Table 3.3. Write an expression for the average power P delivered to the load in terms of Vs, Rs and the load resistance Ri. Complete the entries of Table 8-6 corresponding to the theoretical values. Include the calculations in a separate sheet.

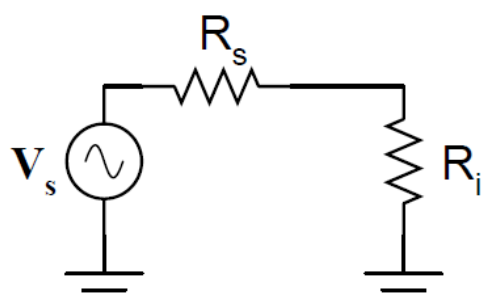

Figure 3.3. Maximum power transfer test circuit.

Parameter	Theoretical	Experimental	Parameter	Theoretical	Experimental
R_S	50 ohm		V_S	2 V	
R_1	27 ohm		P_1		
R_2	56 ohm		P_2		
R_3	100 ohm		P_3		

Table 3.3. Maximum power transfer circuit parameters.

4. Experimental Work

4.1. Assemble the circuit in Figure 3.1 using the component values of Table 3.1. Observe **Vin** (the reference phasor) with channel 2 of the oscilloscope at all times, use channel 1 to observe the other signals. Take measurements to complete the entries of Table 3.2 corresponding to the experimental values. Notice that the magnitude of **Vin** depends on the input frequency and must be measured for both frequencies. Pay attention to the sign of the phase angle.

4.2. Assemble the circuit of Figure 3.2 using the component values of Table 4.1. Measure the actual values of the components and calculate the frequency at which the load impedance is real using the expression derived in the Preliminary Work section. Set the frequency of the signal generator to this value. Take measurements, with and without the capacitor connected to the circuit, to complete the entries of Table 4.1 corresponding to |**V**| (using channel 2), |**I**| (using channel 1 across R1), and the phase between the two of them $\theta_v - \theta_{V_{R1}}$. Notice that the reading from channel 1 is the scaled value of |**I**| by R_1, therefore it must be divided by the actual value of R_1. Calculate pf, P, Q, and S, fill the Table 4.2.

Parameter	**V_S**	**R_S**	**R**	**R_1**	**L**	**C**
Theoretical	10 V	47 ohm	150 ohm	5.6 ohm	1 mH	0.01 µF
Experimental						

Table 4.1. Power calculations test circuit component values.

Parameter	**With C**	**Without C**
pf		
P (W)		
S (VAR)		
Q (VA)		

Table 4.2. Power calculations.

4.3. Assemble the circuit of Figure 3.3 using the component values of Table 3.3. Use a signal frequency of 1 kHz. For each value of Ri measure the RMS voltage across the load and calculate the experimental values of Table 3.3. Determine which of the three resistor values gives the maximum power transfer. Compare this resistor value with the source resistance R_S.