1. Conformal mapping: linear function.
   (a) Let $f(z) = az + b$ with $a \neq 0$. Show that f is analytic on $\mathbb{C}$ and that $f'(z) \neq 0$ for all $z \in \mathbb{C}$. Conclude that f is conformal at every point of $\mathbb{C}$.
   (b) Describe the geometric effect of $f(z) = 2e^{j\pi/4}z+1$ on lines and circles in the complex plane. In particular, determine the images of the line $\operatorname{Im} z = 1$ and of the circle $|z-1| = 2$.

2. Cauchy integral formula I. Let $f(z) = \dfrac{z^2+1}{z-2}$ and let Γ be the positively oriented circle $|z| = 3$.
   (a) Locate all singularities of f and state which of them lie inside Γ.
   (b) Compute $\displaystyle\int_\Gamma \frac{f(z)}{z-1}\,dz$ using Cauchy's integral formula.
   (c) Evaluate $\displaystyle\int_\Gamma \frac{z^2+1}{(z-1)(z-2)}\,dz$ and explain the relation with part (b).

3. Convergence and properties of power series. Let $\displaystyle\sum_{n=0}^{\infty} a_n(z-2)^n$ be a power series that converges at $z = 5$.
   (a) Show that the radius of convergence R satisfies $R \geq 3$.
   (b) Prove that the series converges uniformly on any closed disk $|z-2| \leq r < R$.
   (c) Let $f(z) = \displaystyle\sum_{n=0}^{\infty} a_n(z-1)^n$ with radius of convergence $R > 0$. Show that f is analytic on $|z-1| < R$ and that
   $$f'(z) = \sum_{n=1}^{\infty} n\,a_n(z-1)^{n-1}$$
   has the same radius of convergence R.

4. Taylor series: computation and application.
   (a) Find the Taylor series of $f(z) = \dfrac{1}{1-z}$ about $z_0 = 0$ and state its radius of convergence.
   (b) Using part (a), obtain the Taylor series about $z_0 = 0$ for $g(z) = \dfrac{1}{(1-z)^2}$ by differentiating term by term and give its radius of convergence.
   (c) Use the series for $g(z)$ to compute
   $$\int_0^{1/2} \frac{dx}{(1-x)^2}$$
   by integrating the corresponding real series term by term and summing it.

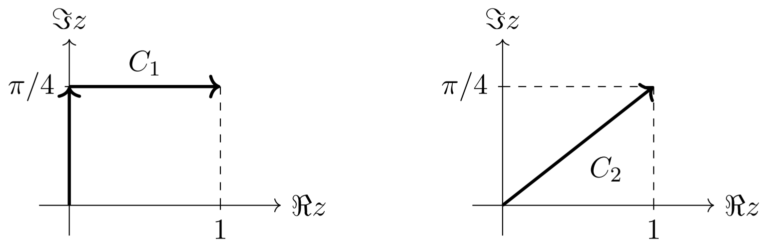

Figure 1: Contours C_1 and C_2 for the following problem.

5. Converting to real integrals: paths C_1 and C_2. Let $f(z) = e^{2z}$. By converting to real integrals, evaluate $\displaystyle\int_{C_1} f(z)\,dz$ and $\displaystyle\int_{C_2} f(z)\,dz$, where the contours C_1 and C_2 are shown in the figure:

- C_1: from 0 vertically to $j\pi/4$, then horizontally to $1 + j\pi/4$,
- C_2: the straight line segment from 0 to $1 + j\pi/4$.