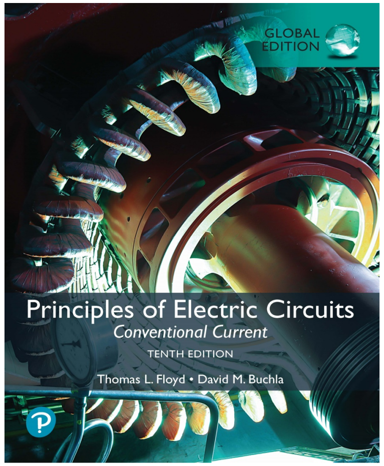
Principles of Electric Circuits: Conventional Current

Tenth Edition, Global Edition



Chapter 9

Branch, Loop, and Node Analyses

Summary: Simultaneous Equations (1 of 2)

Circuit analysis methods in Chapter 9 require use of simultaneous equations.

To simplify solving simultaneous equations, they are usually set up in standard form. Standard form for two equations with two unknowns is

$$a_{1,1}x_1 + a_{1,2}x_2 = b_1$$
$$a_{2,1}x_1 + a_{2,2}x_2 = b_2$$

coefficients → $a_{1,1}$, $a_{2,1}$

variables → x_1, x_2

constants → b_1, b_2

Summary: Simultaneous Equations (2 of 2)

Example:

A circuit has the following equations. Set up the equations in standard form.

$$-10 + 270I_A + 1000(I_A - I_B) = 0$$
$$1000\left(I_B - I_A\right) + 680I_B + 6 = 0$$

Solution:

Rearrange so that variables and their coefficients are in order and put constants on the right.

$$1270I_A - 1000I_B = 10$$
$$-1000I_A + 1680I_B = -6$$

Summary: Solving Simultaneous Equations (1 of 10)

Three methods for solving simultaneous equations are

- Algebraic substitution
- The determinant method
- Using a calculator

Summary: Solving Simultaneous Equations (2 of 10)

Example:

Solve for I_A using **substitution.**

$$1270I_A - 1000I_B = 10$$
$$-1000I_A + 1680I_B = -6$$

Solution:

Solve for I_B in the first equation:

$$I_B = 1.270I_A - 0.010$$

Substitute for I_B into the second equation:

$$-1000I_A + 1680(1.270I_A - 0.010) = -6$$

Rearrange and solve for I_A.

$$1134I_A = 10.8$$
$$I_A = 9.53\text{ mA}$$

Summary: Solving Simultaneous Equations (3 of 10)

If you wanted to find I_B in the previous example, you can substitute the result of I_A back into one of the original equations and solve for I_B. Thus,

$$1270I_A - 1000I_B = 10$$
$$1270(9.53\text{ mA}) - 1000I_B = 10$$
$$I_B = 2.10\text{ mA}$$

Summary: Solving Simultaneous Equations (4 of 10)

The method of **determinants** is another approach to finding the unknowns. The characteristic determinant is formed from the coefficients of the unknowns.

Example:

Write the characteristic determinant for the equations. Calculate its value.

$$1270I_A - 1000I_B = 10$$
$$-1000I_A + 1680I_B = -6$$

Solution:

$$\begin{vmatrix} 1270 & -1000 \\ -1000 & 1680 \end{vmatrix} = 1.134$$

Summary: Solving Simultaneous Equations (5 of 10)

To solve for an unknown by determinants, form the determinant for a variable by substituting the constants for the coefficients of the unknown. Divide by the characteristic determinant.

Constants

Unknown variable

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

Characteristic determinant

To solve for x_2:

$$x_2 = \frac{\begin{pmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{pmatrix}}{\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}}$$

Summary: Solving Simultaneous Equations (6 of 10)

Example:

Solve the same equations using determinants:

$$1270I_A - 1000I_B = 10$$
$$-1000I_A + 1680I_B = -6$$

Solution:

$$I_A = \frac{\begin{vmatrix} 10 & -1000 \\ -6 & 1680 \end{vmatrix}}{\begin{vmatrix} 1270 & -1000 \\ -1000 & 1680 \end{vmatrix}} = 9.53 \text{ mA} \qquad I_B = \frac{\begin{vmatrix} 1270 & 10 \\ -1000 & -6 \end{vmatrix}}{\begin{vmatrix} 1270 & -1000 \\ -1000 & 1680 \end{vmatrix}} = 2.10 \text{ mA}$$

Summary: Solving Simultaneous Equations (7 of 10)

Many scientific calculators allow you to solve simultaneous equations. The specific method depends on your particular calculator, but you will always write the equations in standard form first and then input the number of equations, the coefficients, and the constants. Pressing the *Solve* key will show the values of the unknowns.

$$a_{1,1}x_1 + a_{1,2}x_2 = b_1$$
$$a_{2,1}x_1 + a_{2,2}x_2 = b_2$$

Example:

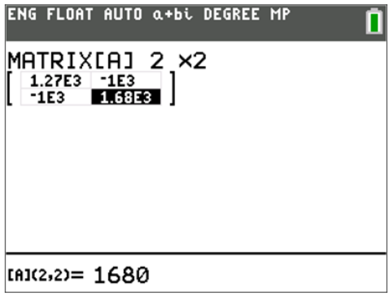
Solve the same equations using the TI-84 Plus CE:

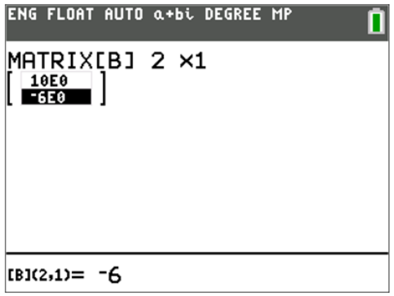
$$1270I_A - 1000I_B = 10$$
$$-1000I_A + 1680I_B = -6$$

Summary: Solving Simultaneous Equations (8 of 10)

Solution:

Press [2nd] [x^{-1}] to open the matrix menu. Choose EDIT and [enter] to enter values into the [A] matrix (the coefficient matrix). Repeat for the [B] matrix. The screens for entering the matrixes are shown:

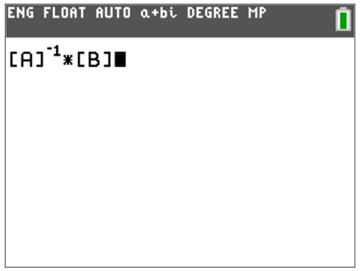




Summary: Solving Simultaneous Equations (9 of 10)

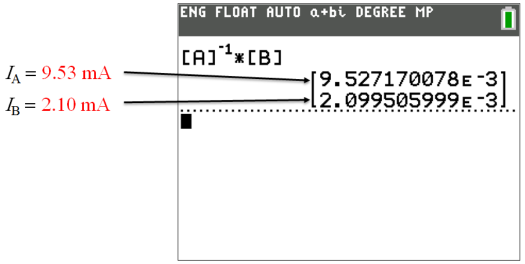
Matrix math is done on the home screen. Press [2nd] [mode] to quit and return to the home screen. Press [2nd] [x^{-1}] and select [A] from the Names menu. Press [x^{-1}] to invert the [A] matrix. Now press [×] [2nd] [x^{-1}] and select [B] from the Names menu. The screen should appear as shown.

Press [enter] to solve.



Summary: Solving Simultaneous Equations (10 of 10)

The solution matrix shows I_A and I_B in amperes. The result confirms the previous calculations.



Note: Some people prefer to solve matrices using a computer program such as Excel to solve matrices using a spreadsheet.

Summary: Branch current method (1 of 3)

In the **branch current method**, you can solve for the currents in a circuit using simultaneous equations.

Steps:

1. Assign a current in each branch in an arbitrary direction.
2. Show polarities according to the assigned directions.
3. Apply KVL in each closed loop.
4. Apply KCL at nodes such that all branches are included.
5. Solve the equations from steps 3 and 4.

Summary: Branch current method (2 of 3)

Example:

Solve for the currents using the branch current method.

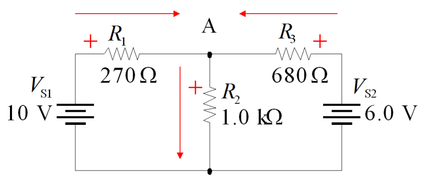
Solution:

5. Solve the equations from steps 3 and 4 (see next slide).

$$0.270I_1 + 1.0I_2 - 10 = 0$$

$$-1.0I_2 - 0.68I_3 + 6.0 = 0$$

$$I_1 + I_3 = I_2$$



Summary: Branch current method (3 of 3)

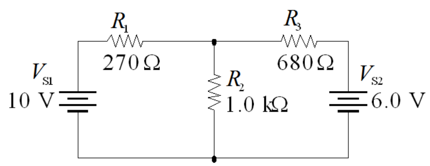
Solution:

In standard form, the equations are

$$\begin{array}{rrrcr} 0.270I_1 & +1.0I_2 & +0 & = & 10 \\ 0 & -1.0I_2 & -0.68I_3 & = & -6.0 \\ I_1 & -I_2 & +I_3 & = & 0 \end{array}$$

The negative result for I_3 indicates the current direction is opposite to the assumed direction.

Solving: I_1 = 9.53 mA, I_2 = 7.43 mA, I_3 = −2.10 mA



Summary: Loop current method (1 of 3)

In the **loop current method**, you can solve for the currents in a circuit using simultaneous equations.

Steps:

1. Assign a current in each nonredundant loop in an arbitrary direction.
2. Show polarities according to the assigned direction of current in each loop.
3. Apply KVL around each closed loop.
4. Solve the resulting equations for the loop currents.

Summary: Loop current method (2 of 3)

Example:

Solve for the currents using the loop current method.

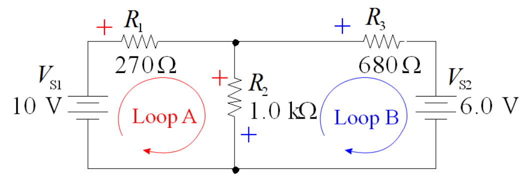
Solution:

4. Solve the resulting equations for the loop currents (see following slide).

$$-10 + 0.270 I_A + 1.0(I_A - I_B) = 0$$

$$1.0(I_B - I_A) + 0.68 I_B + 6.0 = 0$$

Notice that the polarity of R_3. changes depending on the loop.



Summary: Loop current method (3 of 3)

Solution:

Rearranging the loop equations into standard form:

$$1.270 I_A - 1.0 I_B = 10$$

$$-1.0 I_A + 1.68 I_B = -6.0$$

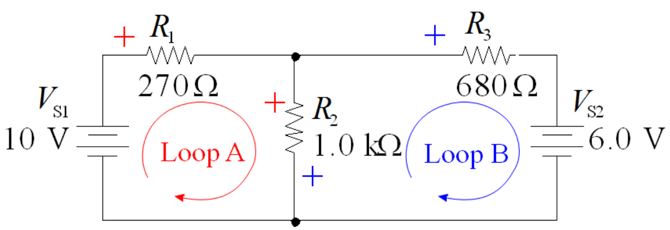
$$I_A = 9.53 \text{ mA}$$

$$I_B = 2.10 \text{ mA}$$

$I_1 = I_A$ = 9.53 mA

$I_2 = I_A - I_B$ = 7.43 mA

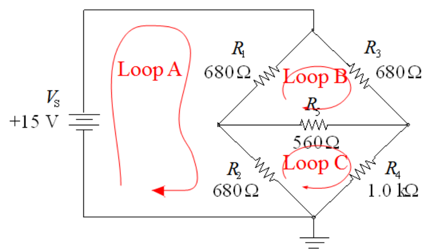
$I_3 = I_B$ = 2.10 mA



Summary: Loop current method applied to circuits with more than two loops

The loop current method can be applied to more complicated circuits, such as the Wheatstone bridge. The steps are the same as shown previously.

The advantage to the loop method for the Wheatstone bridge is that it has only 3 unknowns.

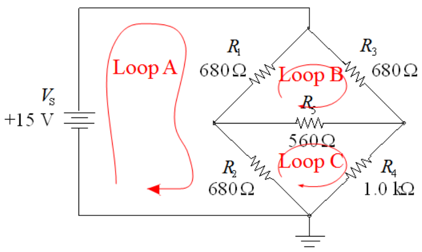


Summary

Example:

Write the loop current equation for Loop A in the Wheatstone bridge:

$$-15+0.68(I_A-I_B)+0.68(I_A-I_C)=0$$



Summary: Node voltage method (1 of 3)

In the **node voltage method**, you can solve for the unknown voltages in a circuit using KCL.

Steps:

1. Determine the number of nodes.
2. Select one node as a reference. Assign voltage designations to each unknown node.
3. Assign currents into and out of each node except the reference node.
4. Apply KCL at each node where currents are assigned.
5. Express the current equations in terms of the voltages and solve for the unknown voltages using Ohm's law.

Summary: Node voltage method (2 of 3)

Example:

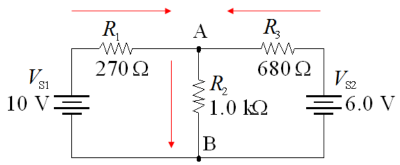
Solve the same problem as before using the node voltage method.

Solution:

5. Write KCL in terms of the voltages (next slide).

$$I_1=\frac{V_{S1}-V_A}{R_1}\quad I_2=\frac{V_A}{R_2}\quad I_3=\frac{V_{S2}-V_A}{R_3}$$

$$I_1+I_3=I_2$$



Summary: Node voltage method (3 of 3)

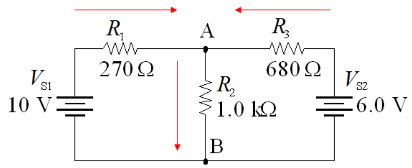
Solution:

$$\frac{V_{S1}-V_A}{R_1}+\frac{V_{S2}-V_A}{R_3}=\frac{V_A}{R_2}\qquad \frac{10-V_A}{0.27}+\frac{6.0-V_A}{0.68}=\frac{V_A}{1.0}$$

$$0.68(10-V_A)+0.27(6.0-V_A)=(0.27)(0.68)V_A=0.184V_A$$

$$-0.68V_A-0.27V_A-0.183V_A=-6.8-1.62$$

$$V_A=7.43\text{ V}$$



Key Terms

Branch One current path that connects two nodes.

Determinant The solution of a square matrix consisting of an array, resulting in a specific value

Loop A closed current path in a circuit.

Matrix An array of numbers.

Node The junction of two or more components.

Simultaneous equations A set of *n* equations containing *n* unknowns, where *n* is a number with a value of 2 or more.

Quiz (1 of 11)

1. In a set of simultaneous equations, the coefficient that is written $a_{1,2}$ appears in

   a. the first equation

   b. the second equation

   c. both of the above

   d. none of the above

Quiz (2 of 11)

2. In standard form, the constants for a set of simultaneous equations are written

   a. in front of the first variable

   b. in front of the second variable

   c. on the right side of the equation

   d. all of the above

Quiz (3 of 11)

3. To solve simultaneous equations, the minimum number of independent equations must be at least

   a. two

   b. three

   c. four

   d. equal to the number of unknowns

Quiz (4 of 11)

4. In the equation $a_{1,1}x_1 + a_{1,2}x_2 = b_1$, the quantity b_1 represents
   a. a constant
   b. a coefficient
   c. a variable
   d. none of the above

Quiz (5 of 11)

5. The value of the determinant $\begin{pmatrix} 3 & 5 \\ 2 & 8 \end{pmatrix}$ is
   a. 4
   b. 14
   c. 24
   d. 34

Quiz (6 of 11)

6. The characteristic determinant for a set of simultaneous equations is formed using
   a. only constants from the equations
   b. only coefficients from the equations
   c. both constants and coefficients from the equations
   d. none of the above

Quiz (7 of 11)

7. A negative result for a current in the branch method means
   a. there is an open path
   b. there is a short circuit
   c. the result is incorrect
   d. the direction of current is opposite to the assumed direction

Quiz (8 of 11)

8. To solve a circuit using the loop method, the equations are first written for each loop by applying
   a. KCL
   b. KVL
   c. Ohm's law
   d. Thevenin's theorem

Quiz (9 of 11)

9. A Wheatstone bridge can be solved using loop equations. The minimum number of nonredundant loop equations required is
   a. one
   b. two
   c. three
   d. four

Quiz (10 of 11)

10. In the node voltage method, the equations are developed by first applying
   a. KCL
   b. KVL
   c. Ohm's law
   d. Thevenin's theorem

Quiz (11 of 11)

Answers:

1. a
2. c
3. d
4. a
5. b
6. b
7. d
8. b
9. c
10. a

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