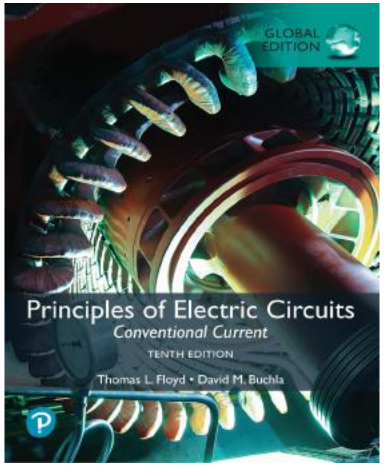
Principles of Electric Circuits: Conventional Current

Tenth Edition, Global Edition



Chapter 19

Circuit Theorems in AC Analysis

Summary: Superposition theorem (1 of 8)

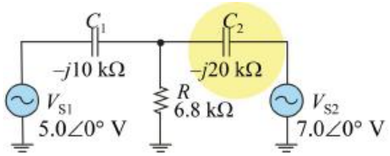
The superposition theorem that you studied in dc circuits can be applied to ac circuits by using complex numbers. Recall that it is applied to circuits with multiple independent sources to solve for the current in any element.

One way to summarize the superposition theorem is

In a linear circuit with multiple independent sources, the current in any element is the algebraic sum of the currents produced by each source acting alone.

Summary: Superposition theorem (2 of 8)

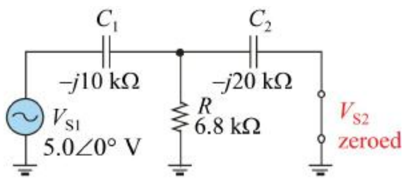
Steps from the text are applied in the following example. To simplify this example, reactances are given for the capacitors.



Example

What is the current in C_2? The internal source resistances are zero.

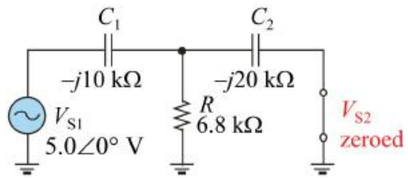
Step 1.
Replace V_{S2} with its internal impedance (zero) and find I_{C2} due to V_{S1} acting alone.



Note: To simplify math equations for this example, units are not shown on the solution. All impedances are in kΩ, currents in mA, and voltages in V.

Summary: Superposition theorem (3 of 8)

Step 1. *(continued)*
Looking from V_{S1}, the total impedance (in kΩ) is



$$\mathbf{Z} = \mathbf{X}_{C1} + \frac{\mathbf{R}\mathbf{X}_{C2}}{\mathbf{R}+\mathbf{X}_{C2}} = -j10 + \frac{(6.8\angle 0^\circ)(20\angle -90^\circ)}{6.8 - j20} = -j10 + \frac{(6.8\angle 0^\circ)(20\angle -90^\circ)}{21.12\angle -71.2^\circ}$$

$$\mathbf{Z} = -j10 + (6.44\angle -18.8^\circ) = -j10 + (6.10 - j2.07) = 6.10 - j12.07 = 13.5\angle -63.2^\circ$$

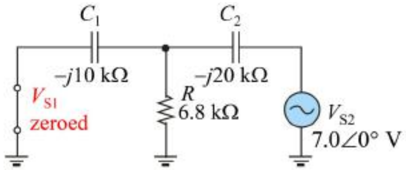
The total current from V_{S1} is $$\mathbf{I}_{S1} = \frac{\mathbf{V}_{S1}}{\mathbf{Z}} = \frac{5.0\angle 0^\circ}{13.5\angle -63.2^\circ} = 0.37\angle 63.2^\circ$$

Apply current divider to find I_{C2} due to V_{S1}:

$$\mathbf{I}_{C2} = \frac{R\angle 0^\circ}{R - jX_{C2}}\mathbf{I}_{S1} = \left(\frac{6.8\angle 0^\circ}{21.12\angle -71.2^\circ}\right)0.37\angle 63.2^\circ = 0.119\angle 134.4^\circ$$

Summary: Superposition theorem (4 of 8)

Step 2.
Find I_{C2} due to V_{S2} by zeroing V_{S1}. The total impedance (in kΩ) is



$$\mathbf{Z} = \mathbf{X}_{C2} + \frac{\mathbf{R}\mathbf{X}_{C1}}{\mathbf{R} + \mathbf{X}_{C1}} = -j20 + \frac{(6.8\angle 0^\circ)(10\angle -90^\circ)}{6.8 - j10} = -j20 + \frac{(6.8\angle 0^\circ)(10\angle -90^\circ)}{12.1\angle -55.8^\circ}$$

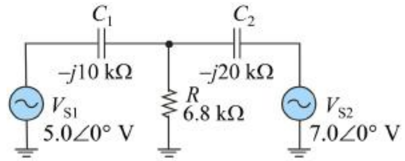
$$= -j20 + (5.62\angle -34.2^\circ) = -j20 + (4.65 - j3.16) = 4.65 - j23.16 = 23.6\angle -78.6^\circ$$

The total current from V_{S2} is $\mathbf{I}_{S2} = \frac{\mathbf{V}_{S2}}{\mathbf{Z}} = \frac{7.0\angle 0^\circ}{23.6\angle -78.6^\circ} = 0.296\angle 78.6^\circ$

Notice that $I_{S2} = I_{C2}$ for this source and is opposite to I_{C2} due to V_{S1}.

Summary: Superposition theorem (5 of 8)

Step 3.
Find I_{C2} due to both sources by combining the results from steps 1 and 2.



I_{C2} due to V_{S1} is $0.119\angle 134.4^\circ = -0.083 + j0.085$

I_{C2} due to V_{S2} is $0.297\angle 78.6^\circ = 0.059 + j0.291$

I_{C2} due to both sources is $-0.142 - j0.206$

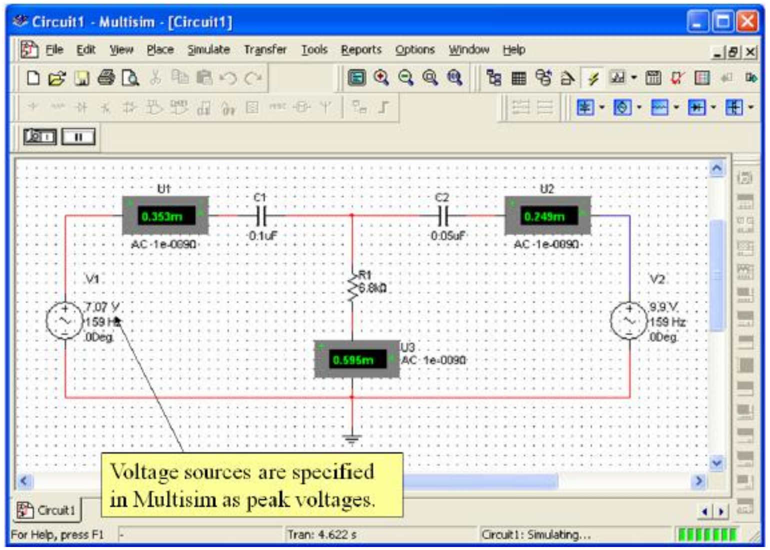
Subtract because currents oppose.

In polar form, the total current in I_{C2} is $0.250\angle -124.6^\circ$ mA

The result is in the 3rd quadrant, meaning that the actual direction is reversed from the assumed positive direction. It is due mainly to V_{S2}.

Problems like this example can be easily solved by ***Multisim****. The Multisim circuit is shown on the following slide.*

Superposition theorem



Summary: Superposition theorem (6 of 8)

Steps for applying the superposition theorem:

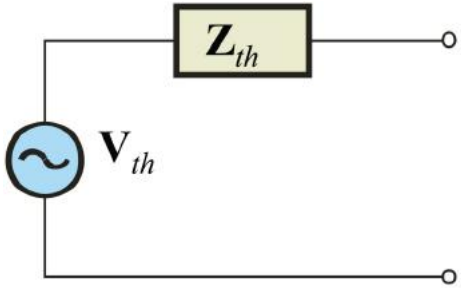
1. Leave one of the sources in the circuit, and replace all others with their internal impedance. For ideal voltage sources, the internal impedance is zero. For ideal current sources, the internal impedance is infinite. We will call this procedure *zeroing* the source.
2. Find the current in the branch of interest produced by the one remaining source.
3. Repeat Steps 1 and 2 for each source in turn. When complete, you will have a number of current values equal to the number of sources in the circuit.
4. Add the individual current values as complex quantities.

Summary: Thevenin's theorem (1 of 5)

Thevenin's theorem can be applied to ac circuits. As applied to ac circuits, Thevenin's theorem can be stated as:

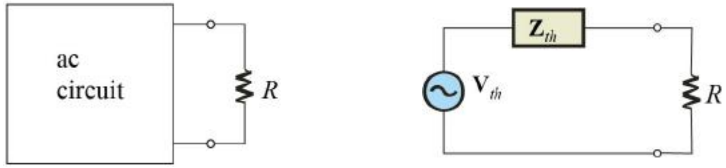
Any two-terminal linear ac circuit can be reduced to an equivalent circuit that consists of an ac voltage source in series with an equivalent impedance.

The Thevenin equivalent for ac circuits is:



Summary: Thevenin's theorem (2 of 5)

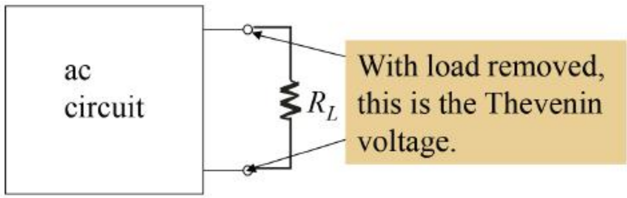
Equivalency means that when the same value of load is connected to both the original circuit and Thevenin's equivalent circuit, the load voltages and currents are the same for both.

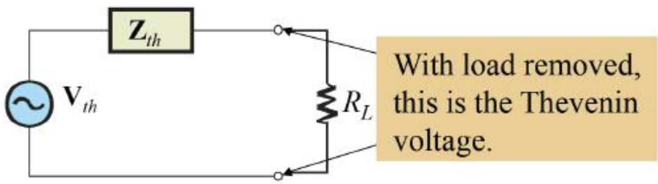


For example, if the same *R* is connected to both circuits, the voltage across *R* will be identical for each circuit.

Summary: Thevenin's theorem (3 of 5)

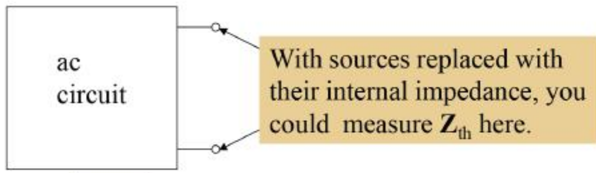
Thevenin's equivalent voltage (V_{th}) is the open-circuit voltage between two specified terminals in a circuit.

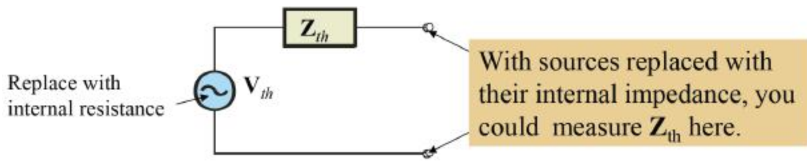




Summary: Thevenin's theorem (4 of 5)

Thevenin's equivalent impedance (Z_{th}) is the total impedance appearing between two specified terminals in a given circuit with all sources replaced by their internal impedances.

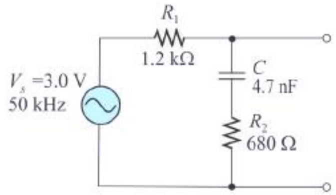




Summary: Thevenin's theorem (5 of 5)

Example

Draw the Thevenin equivalent for the circuit. Assume the internal source resistance is zero.



Solution:

$$\mathbf{Z}_{th} = (X_C + R_2) \parallel R_1 = \left(-j\frac{1}{2\pi(50\text{ kHz})(4.7\text{ nF})} + 680\ \Omega\right) \parallel 1.2\text{ k}\Omega$$

$$= (-j677\ \Omega + 680\ \Omega) \parallel 1.2\text{ k}\Omega = 960\angle -44.9° \parallel 1.2\text{ k}\Omega$$

$$= 522 - j244\ \Omega = 576\angle -25.1°\ \Omega$$

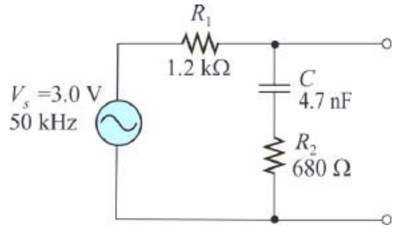
$$\mathbf{V}_{th} = V_s\left(\frac{X_C + R_2}{X_C + R_2 + R_1}\right) = 3.0\text{ V}\left(\frac{-j677 + 680\ \Omega}{-j677 + 680\ \Omega + 1.2\text{ k}\Omega}\right)$$

$$= 3.0\text{ V}\left(\frac{960\angle -44.9°\ \Omega}{2.00\angle -19.8°\text{ k}\Omega}\right) = 1.44\angle -25.1°\text{ V}$$

Summary: Thevenin's theorem

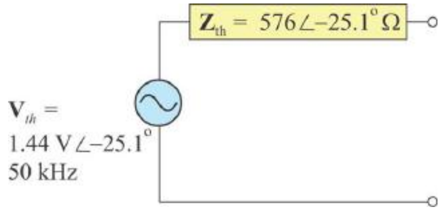
Example

Draw the Thevenin equivalent for the circuit. Assume the internal source resistance is zero.



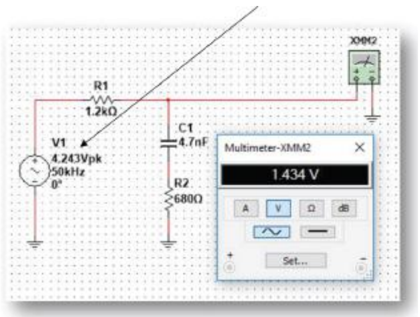
Solution:

Using the results from the previous slide, the Thevenin equivalent can now be drawn:



Summary: Thevenin's theorem (1 of 2)

You can check the magnitude of V_{th} in the previous example by setting up the circuit in Multisim and viewing the unloaded output with the DMM. Multisim uses the peak voltage for setting up the source voltage, so you need to convert it to an equivalent 3.0 V_{rms} for the check.



Summary: Thevenin's theorem (2 of 2)

Steps for applying Thevenin's theorem:

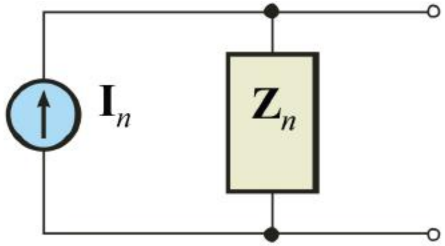
1. Open the two terminals between which you want to find the Thevenin circuit. This is done by removing the component from which the circuit is to be viewed.
2. Determine the voltage across the two open terminals.
3. Determine the impedance viewed from the two open terminals with ideal voltage sources replaced with shorts and ideal current sources replaced with opens.
4. Connect $\mathbf{V}_{th}$ and $\mathbf{Z}_{\text{th}}$ in series to produce the complete Thevenin equivalent circuit.

Summary: Norton's theorem (1 of 7)

Norton's theorem is also an equivalent circuit that provides a way to reduce complicated circuits to a simpler form. As applied to ac circuits, Norton's theorem can be stated as:

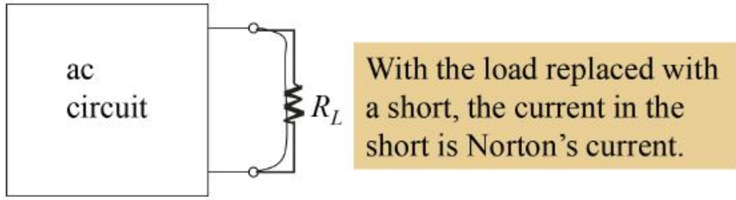
Any two-terminal linear ac circuit can be reduced to an equivalent circuit that consists of an ac current source in parallel with an equivalent impedance.

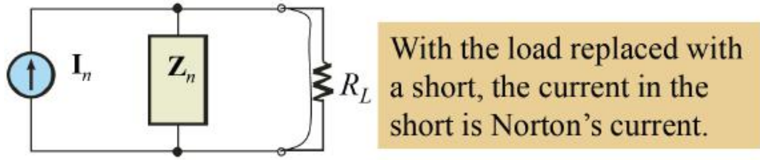
The Norton equivalent for ac circuits is:



Summary: Norton's theorem (2 of 7)

Norton's equivalent current ($\mathbf{I}_n$) is the short-circuit current ($\mathbf{I}_{SL}$) between two specified terminals in a circuit.

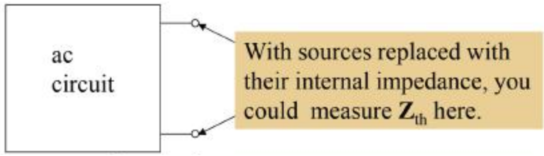


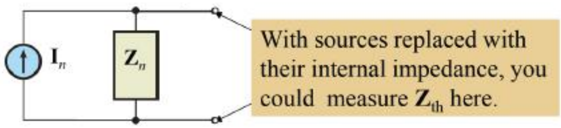


Summary: Norton's theorem (3 of 7)

Norton's equivalent impedance ($\mathbf{Z}_n$) is the total impedance appearing between two specified terminals in a given circuit with all sources replaced by their internal impedances.

An ideal current source is replaced with an open circuit.



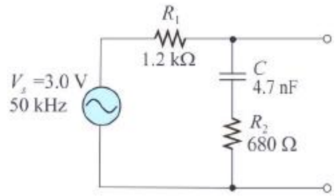


Notice that $\mathbf{Z}_n = \mathbf{Z}_{th}$.

Summary: Norton's theorem (4 of 7)

Example

Draw the Norton equivalent for the circuit that was previously given as an example for Thevenin's equivalent.

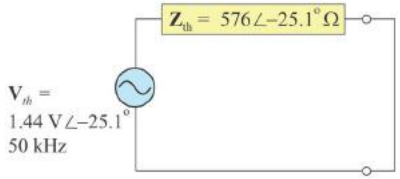


Solution:

The simplest way is to find Norton's equivalent is use the Thevenin circuit previously found. Place a short on the output and calculate the current in the short. From Ohm's law:

$$\mathbf{I}_n = \frac{\mathbf{V}_{th}}{\mathbf{Z}_{th}} = \frac{1.44\angle -25.1^\circ \text{ V}}{576\angle -25.1^\circ \text{ k}\Omega}$$

$$= 2.50\angle 0^\circ \text{ mA}$$

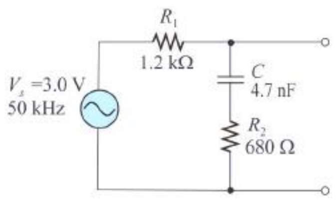


The Norton circuit is shown on the next slide

Summary: Norton’s theorem (5 of 7)

Example

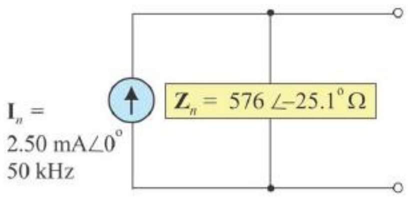
Draw the Norton equivalent for the circuit that was previously given as an example for Thevenin’s equivalent.



Solution:

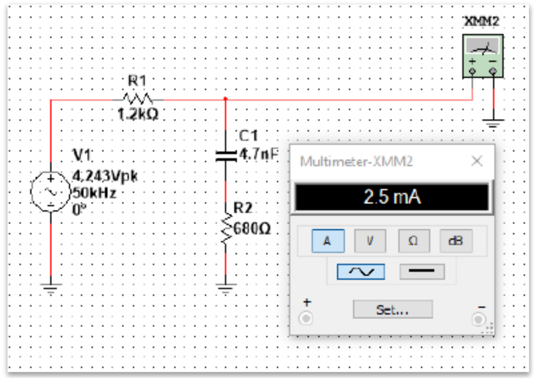
The circuit is:

The Norton current was found in previous calculation. The Norton impedance is the same as the Thevenin impedance.



Summary: Norton’s theorem (6 of 7)

A simple check of the magnitude of I_n in the previous example is by viewing the shorted output current in Multisim using the original circuit. The result confirms the calculation.



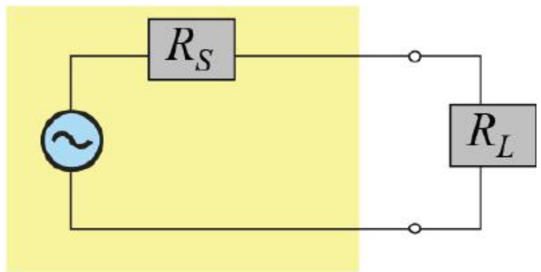
Summary: Norton’s theorem (7 of 7)

Steps for applying Norton’s theorem:

1. Replace the load connected to the two terminals between which the Norton circuit is to be determined with a short.
2. Determine the current through the short. This is $\mathbf{I}_n$.
3. Open the terminals and determine the impedance between the two open terminals with all sources replaced with their internal impedances. This is $\mathbf{Z}_n$.
4. Connect $\mathbf{I}_n$ and $\mathbf{Z}_n$ in parallel.

Summary: Maximum power transfer theorem (1 of 3)

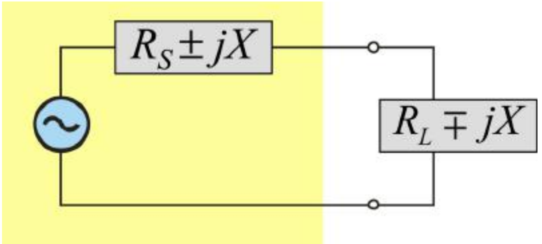
Recall that in resistive circuits, maximum power is transferred from a given source to a load if $R_L = R_S$.



Summary: Maximum power transfer theorem (2 of 3)

Recall that in resistive circuits, maximum power is transferred from a given source to a load if $R_L = R_S$.

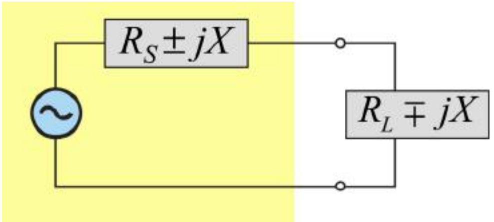
Impedance consists of a resistive and reactive part. To transfer maximum power in reactive circuits, the load resistance should still match the source resistance but the load reactance should cancel the source reactance as illustrated.



Summary: Maximum power transfer theorem (3 of 3)

The impedance that cancels the reactive portion of the source impedance is called the *complex conjugate*. The complex conjugate of *R* - *jX* is *R* + *jX*. Notice that each is the complex conjugate of the other.

In effect the reactance portion of the load forms a series resonant circuit with the reactive portion of the source at the resonant frequency. Thus the matching is frequency dependent.



Selected Key Terms

Complex conjugate A complex number having the same real part and oppositely signed imaginary part; an impedance containing the same resistance and a reactance opposite in phase but equal in magnitude to that of a given impedance.

Equivalent circuit A circuit that produces the same voltage and current to a given load as the original circuit that it replaces.

Quiz (1 of 11)

1. Compared to the original circuit that it replaces, an equivalent circuit will always have
   a. the same power dissipation.
   b. the same number of components
   c. the same voltage and current for a specific load
   d. all of the above

Quiz (2 of 11)

2. To apply the superposition theorem, all sources but the one of interest are replaced with
   a. a short
   b. an open
   c. their internal impedance
   d. an impedance equal to the load

Quiz (3 of 11)

3. Compared to the Thevenin impedance for a given circuit, the Norton impedance is
   a. the same
   b. smaller
   c. larger
   d. equal to the complex conjugate

Quiz (4 of 11)

4. If you put a short on a Thevenin circuit, the current in the short is
   a. zero
   b. smaller than the Norton current
   c. larger than the Norton current
   d. the same as the Norton current

Quiz (5 of 11)

5. A Thevenin circuit is a
   a. voltage source in parallel with an impedance
   b. voltage source in series with an impedance
   c. current source in parallel with an impedance
   d. current source in parallel with an impedance

Quiz (6 of 11)

6. A Norton circuit is a
   a. voltage source in parallel with an impedance
   b. voltage source in series with an impedance
   c. current source in parallel with an impedance
   d. current source in parallel with an impedance

Quiz (7 of 11)

7. A theorem that algebraically adds the current from several sources acting independently to find the total current is
   a. superposition theorem
   b. Thevenin's theorem
   c. Norton's theorem
   d. maximum power theorem

Quiz (8 of 11)

8. The Thevenin equivalent voltage will appear on the output terminals of a circuit if the load impedance is
   a. removed
   b. equal to the source impedance
   c. the complex conjugate of the source impedance
   d. a pure resistance

Quiz (9 of 11)

9. To deliver maximum power to a load from a fixed source impedance, the load should be
   a. identical to the source impedance
   b. the complex conjugate of the source impedance
   c. only the resistive portion of the source impedance
   d. only the reactive portion of the source impedance

Quiz (10 of 11)

10. A 10 kΩ resistor is in series with a capacitor with 10 kΩ reactance. The complex conjugate of this impedance is

    a. 20 kΩ
    b. 10 kΩ + j10 kΩ
    c. 10 kΩ – j10 kΩ
    d. none of the above

Quiz (11 of 11)

Answers:

1. c
2. c
3. a
4. d
5. b
6. c
7. a
8. a
9. b
10. b