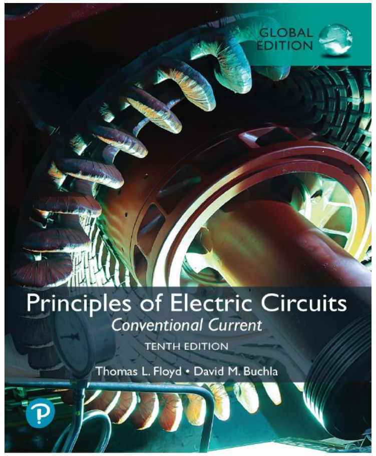
# Principles of Electric Circuits: Conventional Current

Tenth Edition, Global Edition




Chapter 13

RC Circuits



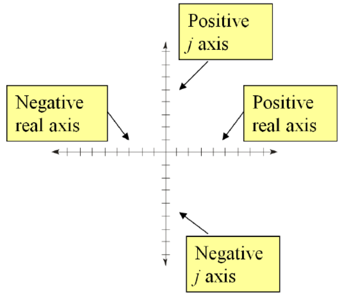
## Summary: Complex numbers (1 of 3)

Recall from Chapter 11 that a rotating vector was defined as a *phasor*. Phasors are useful in analysis of ac circuits.

The **complex plane** is used to plot vectors and phasors.

All **real positive and negative numbers** are plotted along the horizontal axis, which is the real axis.

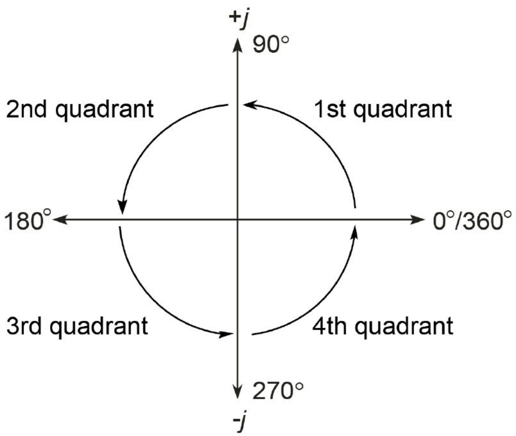
All **imaginary positive and negative numbers** are plotted along the vertical axis, which is the imaginary axis.






## Summary: Complex numbers (2 of 3)

Angular positions can be represented on the complex plane measured from the positive real axis.






## Summary: Complex numbers (3 of 3)

When a point does not lie on an axis, it is a complex number and is defined by its coordinates.

Example

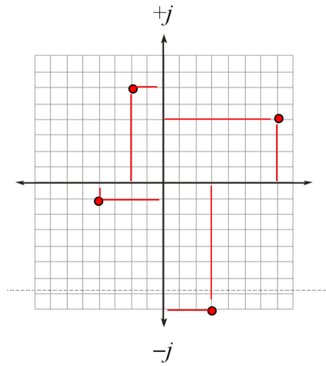
Determine the coordinates for each point.

The point in quadrant 1 is 7, *j*4

The point in quadrant 2 is –2, *j*6

The point in quadrant 3 is –4, –*j*1

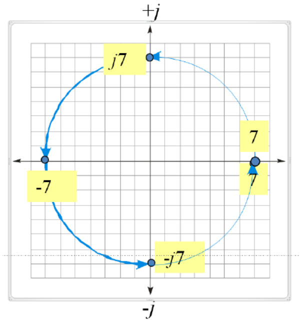
The point in quadrant 4 is 3, –*j*8



# Summary: Value of *j*

*j* has the effect of rotation. A real number, when multiplied by *j* places it on the +*j* axis, effectively rotating it through 90°.

For example, multiply the real number 7 by *j* successively. Notice that each successive multiplication by *j* rotates the phasor by 90°.






# Rectangular form

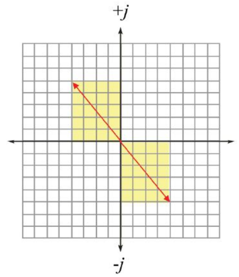
Complex numbers can be expressed in either of two forms: rectangular form or polar form.

The rectangular form describes a phasor as the sum of the real value (*A*) and the imaginary value (*jB*): A + *jB*

For example, the phasor shown is written in rectangular form as –4 + j5

Question:

Where is the phasor +4 –*j*5 plotted?






# Polar form

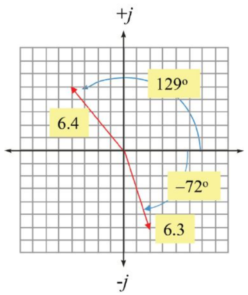
The polar form describes a phasor in terms of a magnitude (*C*) and angular position ( θ) relative to the positive real (*x*) axis.

$$C\angle \pm\theta$$

For example, the phasor shown is written in polar form as $6.4\ \angle\ 129°$

Question:

Where is the phasor $6.3\ \angle -72°$ plotted?





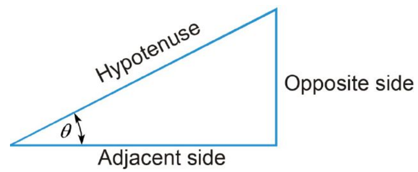

# Conversion from Rectangular to Polar Form

Basic trig functions, as well as the Pythagorean theorem allow you to convert between rectangular and polar notation and vice-versa. Reviewing these relationships:

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$




$$\text{hypotenuse}^2 = \text{adjacent side}^2 + \text{opposite side}^2$$

# Summary: Conversion from Rectangular to Polar Form (1 of 2)

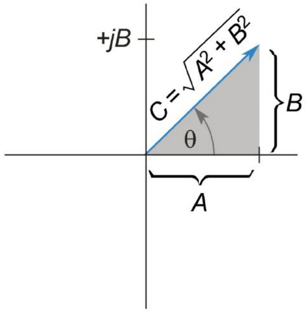
Converting from rectangular form ($A + jB$), to polar form is

$(C\angle \pm\theta)$ is done as follows:

$$C = \sqrt{A^2 + B^2}$$

and

$$\theta = \tan^{-1}\frac{\pm B}{A}$$




## Example

Convert –5.0 + $j$3.3 to polar form. Notice that the result will be in the second quadrant.

$$C = \sqrt{A^2 + B^2} = \sqrt{-5.0^2 + 3.3^2} = 5.99$$

$$\theta = \tan^{-1}\frac{\pm B}{A} = \tan^{-1}\frac{3.3}{-5.0} = 180° - 33° = 147°$$

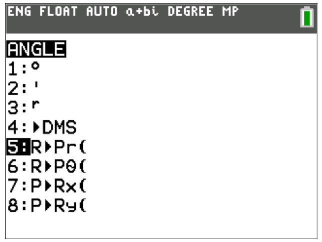
$5.99\angle 147°$

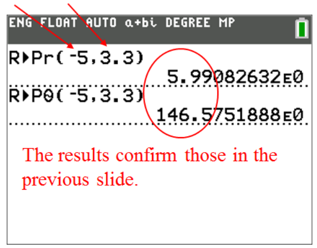


# Summary: Conversion from Rectangular to Polar Form (2 of 2)

All scientific calculators can perform the conversions. The TI-84 Plus CE has built-in conversion in the angle menu Press 2nd apps to enter the angle menu.

From the angle menu, press 5 on the keyboard for the magnitude or 6 for the angle. Let's convert –5.0 + $j$3.3 to polar form again.





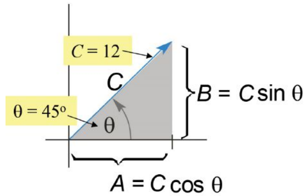

# Summary: Conversion from Polar to Rectangular Form (1 of 2)

Converting from polar form $(C\angle \pm\theta)$ to rectangular form ($A + jB$),is done as follows:

$$A = C\cos\theta$$

and

$$B = C\sin\theta$$




## Example

Convert $12\angle 45°$ to rectangular form.

$$A = C\cos\theta = 12\cos 45° = 8.49$$

$$B = C\sin\theta = 12\sin 45° = 8.49$$

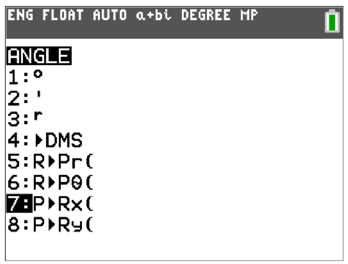
8.49 +$j$8.49

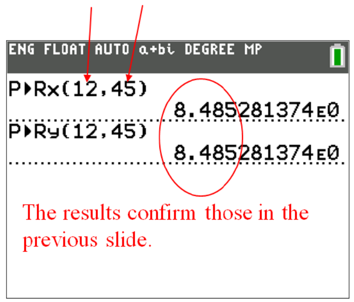


# Summary: Conversion from Polar to Rectangular Form (2 of 2)

Again, the TI-84 Plus CE has built-in can do the conversion using the angle menu. Press 2nd apps to enter the angle menu and press 7 on the keyboard for the magnitude and 8 for the angle.

The conversion of $12\angle 45°$ is illustrated



## Summary: Mathematical operations (1 of 4)

Complex numbers can be added by putting them in rectangular form first. Then add the real parts of each number to get the real part of the sum. Then add the *j* part of each number to get the *j* part of the sum.

Example

Add 8.48 + j8.48 to 6.20 – *j*5.70.

$$\begin{array}{r} 8.48 + j8.46 \\ 6.20 - j5.70 \\ \hline 14.68 + j2.76 \end{array}$$



## Summary: Mathematical operations (2 of 4)

Subtraction is similar to addition. To subtract complex numbers, you can subtract the real parts and the *j* parts separately.

Example

Subtract 6.20 – *j*5.70 from 8.48 + *j*8.48.

$$\begin{array}{r} 8.48 + j8.46 \\ 6.20 - j5.70 \\ \hline 2.28 + j14.16 \end{array}$$



## Summary: Mathematical operations (3 of 4)

Multiplication can be done in either rectangular form or polar form. Generally, polar form is more convenient. To multiply in polar form, multiply the magnitudes and add the angle algebraically.

Example

Multiply by $8.30\angle 25°$ by $12\angle -15°$

$$\left.\begin{array}{r} 8.30 \times 12 = 99.6 \\ \hline 25° + (-15°) = 10° \end{array}\right\} 99.6\angle 10°$$



## Summary: Mathematical operations (4 of 4)

Division can be also be done in either rectangular form or polar form. To divide in polar form, divide the magnitudes and subtract algebraically the angle of the denominator from the angle of the numerator.

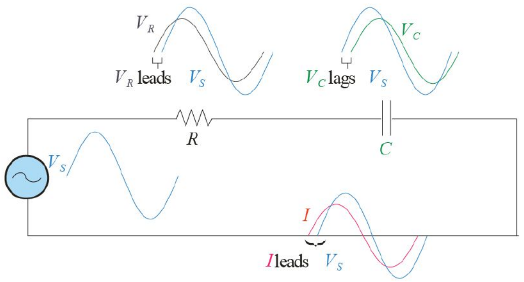
Example

Divide $8.30\angle 25°$ by $12\angle -15°$

$$\left.\begin{array}{r} 8.30 \div 12 = 0.692 \\ \hline 25° - (-15°) = 40° \end{array}\right\} 0.692\angle 40°$$

# Summary: Sinusoidal response of series *RC circuits*

When both resistance and capacitance are in a series circuit, the phase angle between the applied voltage and total current is between 0°and 90°, depending on the values of resistance and reactance.






# Summary: Impedance of series *RC* circuits (1 of 2)

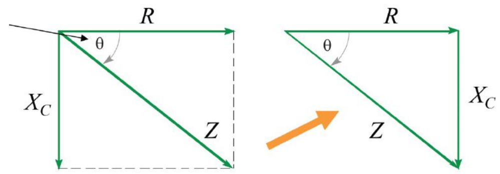
In a series *RC* circuit, the total impedance is the phasor sum of *R* and $-jX_C$.

R is plotted along the positive *x*-axis.

$X_C$ is plotted along the negative *y*-axis (−*j*).

$$\theta = \tan^{-1}\left(\frac{X_C}{R}\right)$$

*Z* is the diagonal




It is convenient to reposition the phasors into the *impedance triangle.*



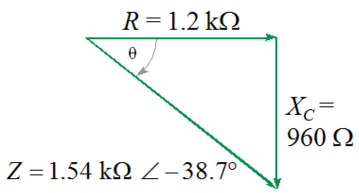
# Summary: Impedance of series *RC* circuits (2 of 2)

## Example

Sketch the impedance triangle and show the values for $R$ = 1.2 k***Ω*** and $X_C$ = 960 ***Ω***.

$$Z = \sqrt{(1.2\ \text{k}\Omega)^2 + (0.96\ \text{k}\Omega)^2} = 1.54\ \text{k}\Omega$$

$$\theta = \tan^{-1}\frac{-0.96\ \text{k}\Omega}{1.2\ \text{k}\Omega} = -38.7°$$






# Summary: Analysis of series *RC* circuits (1 of 2)

Ohm’s law is applied to series *RC* circuits using phasor quantities of **Z**, **V**, and **I**.

$$\mathbf{V} = \mathbf{IZ} \qquad \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} \qquad \mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}}$$

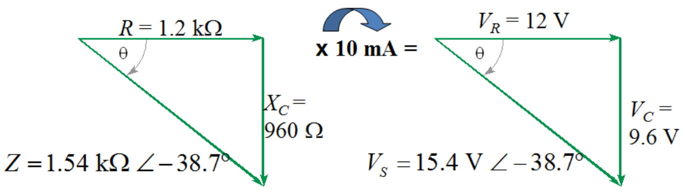
Because *I* is the same everywhere in a series circuit, you can obtain the voltage phasors by simply multiplying the impedance phasors by the current.

## Summary: Analysis of series *RC* circuits (2 of 2)

### Example

Assume the current in the previous example is 10 $mA_{rms}$. Sketch the voltage phasors. The impedance triangle from the previous example is shown for reference.

The voltage phasors can be found from Ohm's law. Multiply each impedance phasor by 10 mA

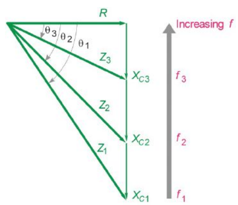





## Summary: Variation of phase angle with frequency

Phasor diagrams that have reactance phasors can be drawn only for a single frequency because $X_C$ is a function of frequency.

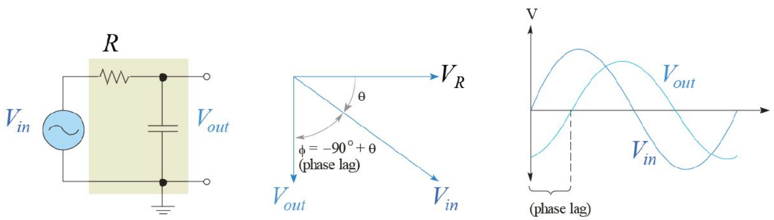
As frequency changes, the impedance triangle for an *RC* circuit changes as illustrated here because $X_C$ decreases with increasing *f*. This determines the frequency response of *RC* circuits.






## Summary: Applications (1 of 3)

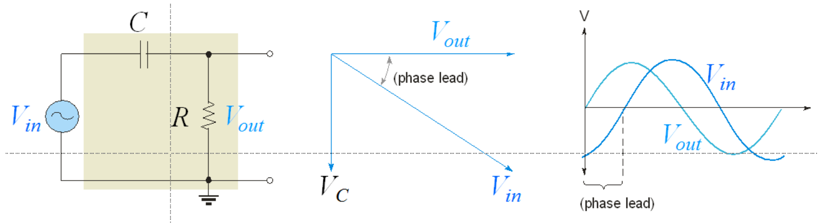
For a given frequency, a series *RC* circuit can be used to produce a phase lag by a specific amount between an input voltage and an output by taking the output across the capacitor. This circuit is also a basic low-pass *filter*, a circuit that passes low frequencies and rejects all others.





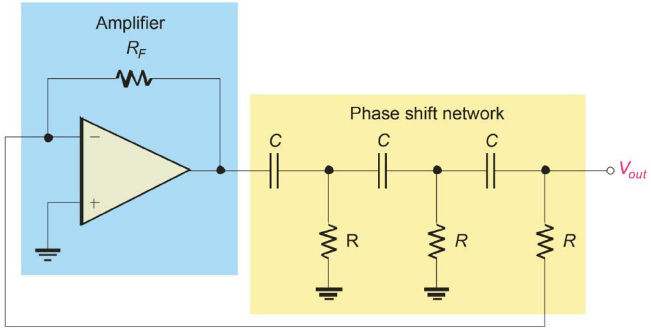

## Summary: Applications (2 of 3)

Reversing the components in the previous circuit produces a circuit that is a basic lead network. This circuit is also a basic high-pass *filter*, a circuit that passes high frequencies and rejects all others.



# Summary: Applications (3 of 3)

An application showing how the phase shift network is useful is the phase-shift oscillator, which uses a combination of *RC* networks to produce the required 180° phase shift for the oscillator.






# Summary: Sinusoidal response of parallel *RC* circuits (1 of 4)

For parallel circuits, it is useful to introduce two new quantities (susceptance and admittance) and to review conductance.

**Conductance** is the reciprocal of resistance. $\mathbf{G} = \frac{1}{\mathbf{R}} = \frac{1}{R\angle 0°}$

**Capacitive susceptance** is the reciprocal of capacitive reactance. $\mathbf{B}_C = \frac{1}{\mathbf{X}_C}$

**Admittance** is the reciprocal of impedance. $\mathbf{Y} = \frac{1}{\mathbf{Z}}$

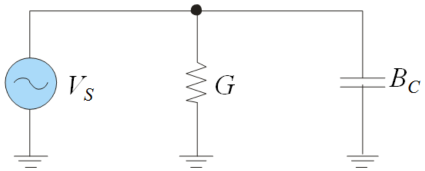


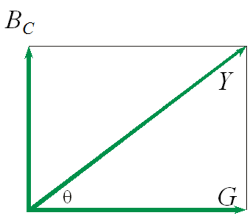
# Summary: Sinusoidal response of parallel *RC* circuits (2 of 4)

In a parallel *RC* circuit, the admittance phasor is the sum of the conductance and capacitive susceptance phasors.

The magnitude can be expressed as $Y = \sqrt{G^2 + B_C^2}$

From the diagram, the phase angle is $\theta = \tan^{-1}\left(\frac{B_C}{G}\right)$






# Summary: Sinusoidal response of parallel *RC* circuits (3 of 4)

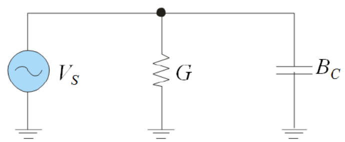
Some important points to notice are:

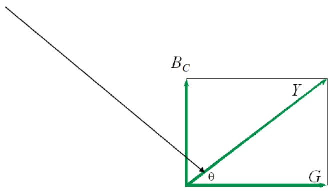
*G* is plotted along the positive *x-axis*.

$B_C$ is plotted along the positive *y-axis* (+*j*).

$$\theta = \tan^{-1}\left(\frac{B_C}{G}\right)$$

*Y* is the diagonal



# Summary: Sinusoidal response of parallel *RC* circuits (4 of 4)

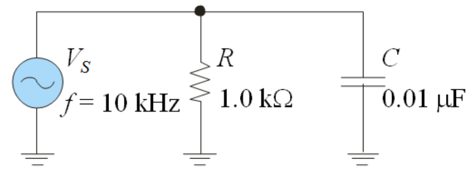
Example

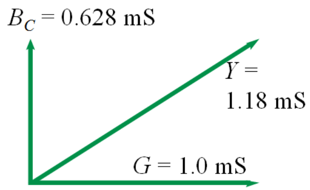
Draw the admittance phasor diagram for the circuit

The magnitude of the conductance and susceptance are:

$$G = \frac{1}{R} = \frac{1}{1.0\ \text{k}\Omega} = 1.0\ \text{mS} \quad B_C = 2\pi\,(10\ \text{kHz})(0.01\ \mu\text{F}) = 0.628\ \text{mS}$$

$$Y = \sqrt{G^2 + B_C{}^2} = \sqrt{(1.0\ \text{mS})^2 + (0.628\ \text{mS})^2} = 1.18\ \text{mS}$$




Pearson



# Summary: Analysis of parallel *RC* circuits (1 of 2)

Ohm's law is applied to parallel *RC* circuits using phasor quantities of **Y**, **V**, and **I**.

$$\mathbf{Y} = \frac{\mathbf{I}}{\mathbf{V}} \qquad \mathbf{V} = \frac{\mathbf{I}}{\mathbf{Y}} \qquad \mathbf{I} = \mathbf{VY}$$

Because *V* is the same across all components in a parallel circuit, you can obtain the current phasors by simply multiplying the admittance phasors by the voltage.

Pearson

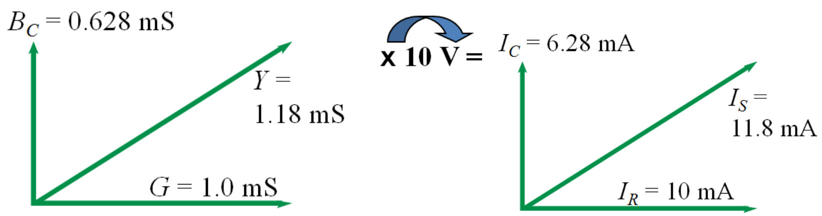


# Summary: Analysis of parallel *RC* circuits (2 of 2)

Example

Assume the voltage in the previous example is 10 V. Sketch the current phasors. The admittance diagram from the previous example is shown for reference.

The current phasors can be found from Ohm's law. Multiply each admittance phasor by 10 V.



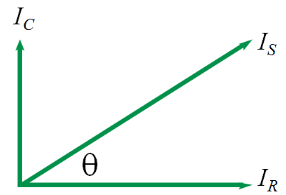

Pearson



# Summary: Phase angle of parallel *RC* circuits

Notice that the formula for capacitive susceptance is the reciprocal of capacitive reactance. Thus $B_C$ and $I_C$ are directly proportional to $B_C = 2\pi fC$

As frequency increases, $B_C$ and $I_C$ must also increase, so the angle between $I_R$ and $I_S$ must increase.

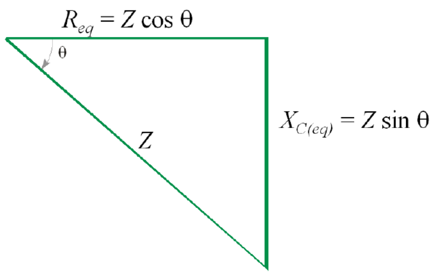



Pearson

# Summary: Equivalent series and parallel *R C* circuits

For every parallel *RC* circuit there is an equivalent series *RC* circuit at a given frequency.

The equivalent resistance and capacitive reactance are shown on the impedance triangle:





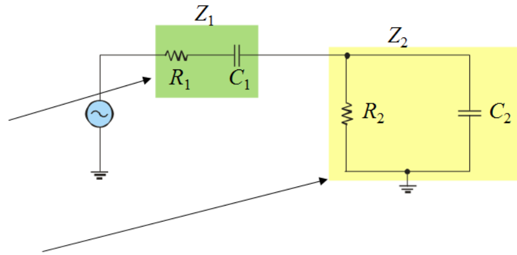


# Summary: Series-Parallel *R C* circuits

Series-parallel *RC* circuits are combinations of both series and parallel elements. The solution of these circuits is similar to resistive combinational circuits except complex numbers must be employed.

For example, the components in the green box are in series:

$$Z_1 = R_1 + XC_1$$




The components in the yellow box are in parallel:

$$Z_2 = \frac{R_2 X_{C2}}{R_2 + X_{C2}}$$

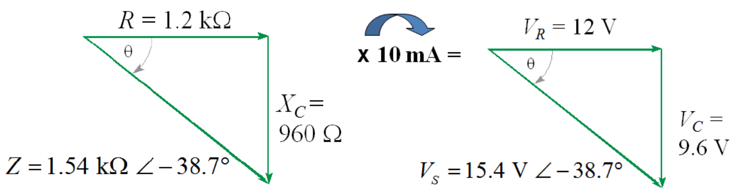
Using phasor math, (and keeping track of angles) you can find the total impedance:

$$Z_T = Z_1 + Z_2$$




# Summary: The Power triangle (1 of 2)

Recall that in a series *R C* circuit, you could multiply the impedance phasors by the current to obtain the voltage phasors. The earlier example is shown for review:





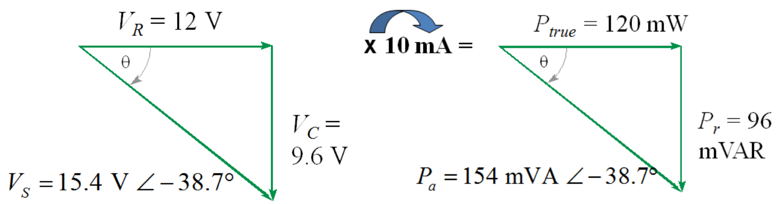


# Summary: The Power triangle (2 of 2)

Multiplying the voltage phasors by $I_{rms}$ gives the power triangle (equivalent to multiplying the impedance phasors by $I^2$). Apparent power is the product of the magnitude of the current and magnitude of the voltage and is plotted along the hypotenuse of the power triangle.

## Example

The rms current in the earlier example was 10 mA. Show the power triangle.



## Summary: Power factor

The power factor is the relationship between the apparent power in volt-amperes and true power in watts. Volt-amperes multiplied by the power factor equals true power.

Power factor is defined mathematically as

$$PF = \cos \theta$$

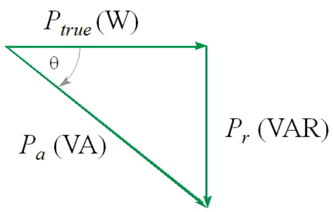
The power factor can vary from 0 for a purely reactive circuit to 1 for a purely resistive circuit.



## Summary: Apparent power

Apparent power consists of two components; a true power component, that does the work, and a reactive power component, that is simply power shuttled back and forth between source and load.

Some components such as transformers, motors, and generators are rated in VA rather than watts.






## Selected Key Terms (1 of 3)

***Complex plane*** An area consisting of four quadrants on which a quantity containing both magnitude and direction can be represented.

***Real number*** A number that exists on the horizontal axis of the complex plane.

***Imaginary number*** A number that exists on the vertical axis of the complex plane.

***Rectangular form*** One form of a complex number made up of a real part and an imaginary part.



## Selected Key Terms (2 of 3)

***Polar form*** One form of a complex number made up of a magnitude and an angle.

***Impedance*** The total opposition to sinusoidal current expressed in ohms.

***Capacitive suceptance ($B_C$)*** The ability of a capacitor to permit current; the reciprocal of capacitive reactance. The unit is the siemens (S).

## Selected Key Terms (3 of 3)

***Power factor*** The relationship between volt-amperes and true power or watts. Volt-amperes multiplied by the power factor equals true power.

***Filter*** A type of circuit that passes certain frequencies and rejects all others.

***Frequency response*** In electric circuits, the variation of the output voltage (or current) over a specified range of frequencies.




## Quiz (1 of 11)

1. Complex numbers can be expressed in polar form. The angle is measured from the

   a. positive real axis
   b. negative real axis
   c. positive imaginary axis
   d. negative imaginary axis




## Quiz (2 of 11)

2. If a phasor that is expressed in polar form has an angle of –45°, it is in the

   a. first quadrant
   b. second quadrant
   c. third quadrant
   d. fourth quadrant




## Quiz (3 of 11)

3. To multiply two numbers that are in polar form,

   a. add the magnitudes and add the angles
   b. multiply the magnitudes and add the angles
   c. add the magnitudes and multiply the angles
   d. multiply the magnitudes and multiply the angles

## Quiz (4 of 11)

4. Given the impedance phasor diagram of a series RC circuit, you could obtain the voltage phasor diagram by

   a. multiplying each phasor by the current
   b. multiplying each phasor by the source voltage
   c. dividing each phasor by the source voltage
   d. dividing each phasor by the current



## Quiz (5 of 11)

5. If you increase the frequency in a series *RC* circuit,

   a. the total impedance will increase
   b. the reactance will not change
   c. the phase angle will decrease
   d. none of the above



## Quiz (6 of 11)

6. In a parallel *RC* circuit, the capacitive susceptance is plotted on an admittance phasor diagram along the

   a. positive real axis
   b. negative real axis
   c. positive imaginary axis
   d. negative imaginary axis



## Quiz (7 of 11)

7. Given the admittance phasor diagram of a parallel *RC* circuit, you could obtain the current phasor diagram by

   a. multiplying each phasor by the voltage
   b. multiplying each phasor by the total current
   c. dividing each phasor by the voltage
   d. dividing each phasor by the total current

## Quiz (8 of 11)

8. If you increase the frequency in a parallel *RC* circuit,
   - a. the total admittance will decrease
   - b. the total current will not change
   - c. the phase angle between $I_R$ and $I_S$ will decrease
   - d. none of the above



## Quiz (9 of 11)

9. The magnitude of the admittance in a parallel *RC* circuit will be larger if
   - a. the resistance is larger
   - b. the capacitance is larger
   - c. both a and b
   - d. none of the above



## Quiz (10 of 11)

10. The maximum power factor occurs when the
    - a. circuit is entirely reactive
    - b. reactive and true power are equal
    - c. circuit is entirely resistive
    - d. product of voltage and current are maximum



## Quiz (11 of 11)

Answers:

1. a
2. d
3. b
4. a
5. c
6. c
7. a
8. d
9. b
10. c