AI-C 205 Probability and Statistics

Homework 5
(Deadline: 10^{th} Nov, 2026)

1. We toss a coin ten times and get the following sample where 1 denotes heads and 0 denotes tails:
$$\{1, 0, 1, 1, 0, 0, 0, 0, 0, 0\}.$$
What are the best predictors in terms of the MSE and the MAE?

2. Imagine a streaming data scenario where we observe X^t one at a time and we want to update the sample mean and sample variance as we see one instance at a time. Let us say M_N and S_N^2 denote the sample mean and variance calculated on the first N instances; derive recursive equations for M_{N+1} and S_{N+1}^2 in terms of M_N , S_N^2, and the next observed X^{N+1}.

3. Show that S_o^2 is an unbiased estimator for σ^2.

4. Show that the sample mean minimizes the empirical MSE:
$$MSE = \sum_{t=1}^{N} (X^t - c)^2.$$

5. Show that the sample median minimizes the empirical MAE:
$$MAE = \sum_{t=1}^{N} |X^t - c|.$$

6. Let us say for the same population parameter θ, we have two independent unbiased estimators d_1 and d_2 with variances σ_1^2 and σ_2^2, respectively. We want to define a new estimator from these two in the form
$$d = \omega d_1 + (1–\omega) d_2.$$
What should ω be so that d is also unbiased and has minimum variance?