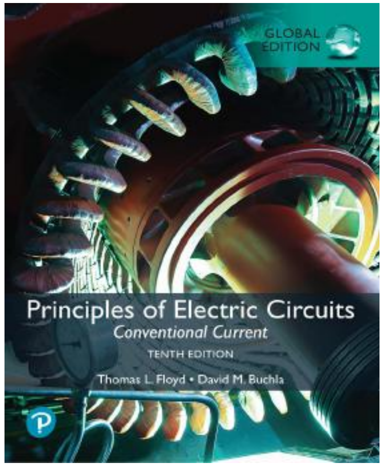
Principles of Electric Circuits: Conventional Current

Tenth Edition, Global Edition



Chapter 17

RLC Circuits and Resonance

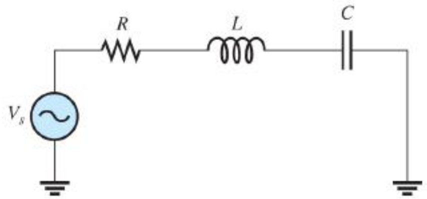
Summary: Impedance of series *RLC* circuits (1 of 10)

When *R*, *L*, and *C* are in a series circuit, the reactance of the inductor and reactance of the capacitor tend to offset each other, depending on the values. The total reactance is

$$X_{tot} = |X_L - X_C|$$

When $X_L > X_C$, the circuit is predominantly inductive.

When $X_C > X_L$, the circuit is predominantly capacitive.



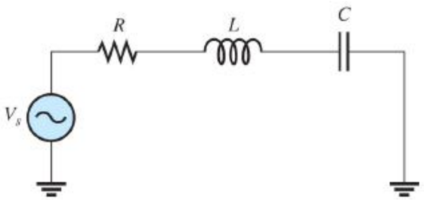
Summary: Impedance of series *RLC* circuits (2 of 10)

The total impedance for the *RLC* circuit is given by

$$\mathbf{Z} = R + jX_L - jX_C$$

In polar form, this is written

$$\mathbf{Z} = \sqrt{R^2 + (X_L - X_C)^2} \angle \tan^{-1} \frac{(X_L - X_C)}{R}$$



Summary: Impedance of series *RLC* circuits (3 of 10)

Note: The circuit here will be used in several more slides.

Example

What is the total impedance for the circuit?

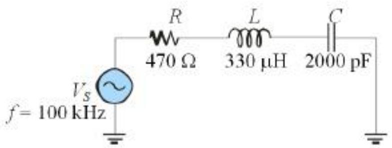
$$X_L = 2\pi fL = 2\pi(100\text{ kHz})(330\ \mu\text{H}) = 207\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(100\text{ kHz})(2000\text{ pF})} = 796\ \Omega$$

$$\mathbf{Z} = R + jX_L - jX_C = 470\ \Omega + j207\ \Omega - j796\ \Omega = 470\ \Omega - j588\ \Omega$$

In polar form,

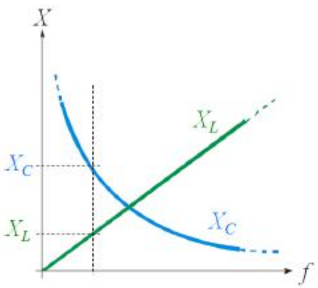
$$\mathbf{Z} = 753\ \Omega\angle -51.4°$$



Summary: Impedance of series *RLC* circuits (4 of 10)

Depending on the frequency, the circuit can appear to be capacitive or inductive. The circuit in the example was capacitive because

$$X_C > X_L$$



Summary: Impedance of series *RLC* circuits (5 of 10)

Example

What is the total impedance for the circuit when the frequency is increased to 400 Hz?

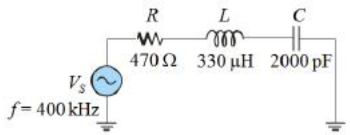
$$X_L = 2\pi fL = 2\pi(400\text{ kHz})(330\ \mu\text{H}) = 829\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(400\text{ kHz})(2000\text{ pF})} = 199\ \Omega$$

$$\mathbf{Z} = R + jX_L - jX_C = 470\ \Omega + j829\ \Omega - j199\ \Omega = 470\ \Omega + j630\ \Omega$$

In polar form,

$$\mathbf{Z} = 786\ \Omega\angle 53.3°$$

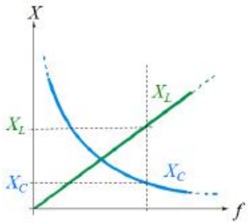


The circuit is now inductive.

Summary: Impedance of series *RLC* circuits (6 of 10)

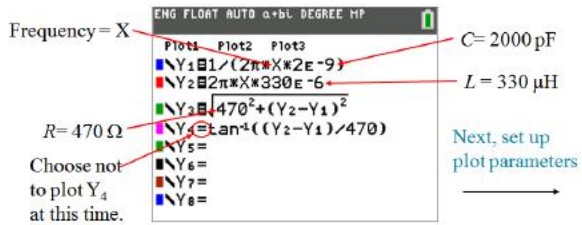
By changing the frequency, the circuit now appears to be inductive because $X_L > X_C$

The frequency response of the series *RLC* circuit in the previous slides can be plotted with the aid of a graphing calculator. The next slide illustrates the steps for plotting the curves here using the TI-84 Pro CE.

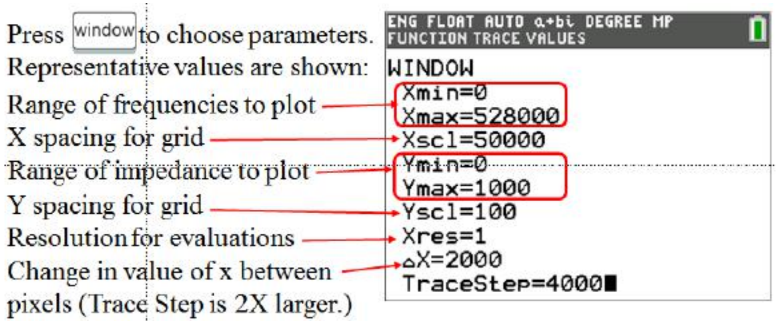


Summary: Impedance of series *RLC* circuits (7 of 10)

Input the equations for X_C (Y_1), X_L(Y_2), Z(Y_3) and θ (Y_4) as shown. The frequency is the independent variable, *X*. To reuse Y_1 and Y_2 in the equation for Z, press vars (variables); select Y-vars and Function from the menu and select the variable you want. Press enter and you will see the selected variable inserted in your equation.



Summary: Impedance of series *RLC* circuits (8 of 10)

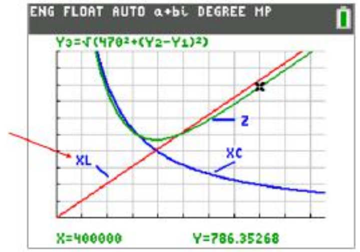


Note: If you choose the value of Xmin and ΔX, the calculator will automatically choose Xmax.

Summary: Impedance of series *RLC* circuits (9 of 10)

Press graph to plot the three selected curves. You can choose to view any of the calculated points by pressing trace. The cursor is shown on the impedance curve at 400 kHz.

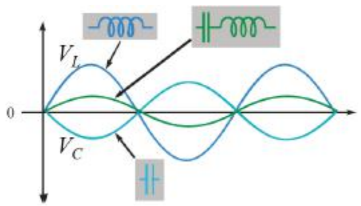
Labels can be added using the draw menu. Press 2nd prgm to access it.



Summary: Voltages in a series *RLC* circuits

The voltages across the *RLC* components must add to the source voltage in accordance with KVL. Because of the opposite phase shift due to *L* and *C*, V_L and V_C effectively subtract.

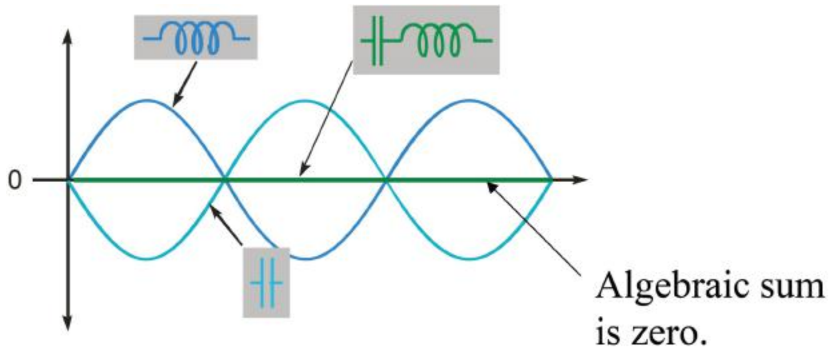
Notice that V_C is out of phase with V_L. When they are algebraically added, the result is….



This example is inductive.

Summary: Series resonance (1 of 5)

At series resonance, X_C and X_L cancel. V_C and V_L also cancel because the voltages are equal and opposite. The circuit is purely resistive at resonance.



Summary: Series resonance (2 of 5)

The formula for resonance can be found by setting $X_C = X_L$. The result is

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

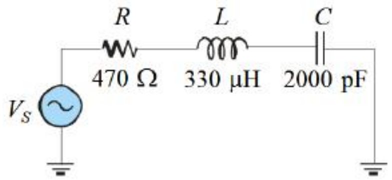
Example

What is the resonant frequency for the circuit?

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

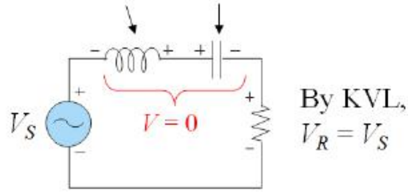
$$= \frac{1}{2\pi\sqrt{(330\quad)(\quad)}}$$

$= 196$ kHz



Summary: Series resonance (3 of 5)

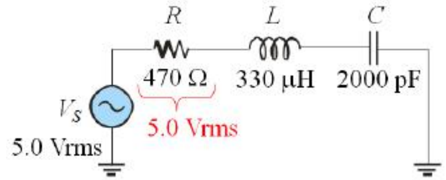
Ideally, at resonance the sum of V_L and V_C is zero.



Example

What is V_R at resonance?

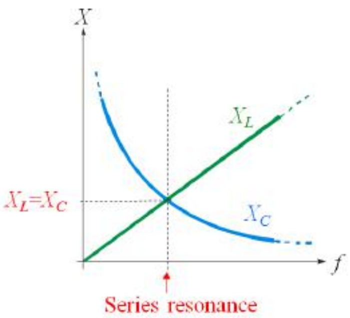
5.0 Vrms



Summary: Impedance of series *RLC* circuits (10 of 10)

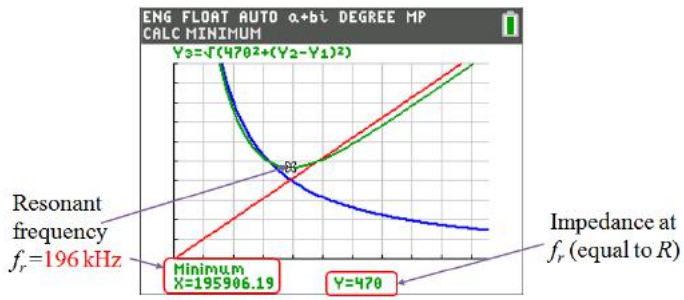
At the series resonant frequency, $X_C = X_L$.

At series resonance, the total impedance is only due to the resistance of the circuit because of the cancellation of opposite phases.



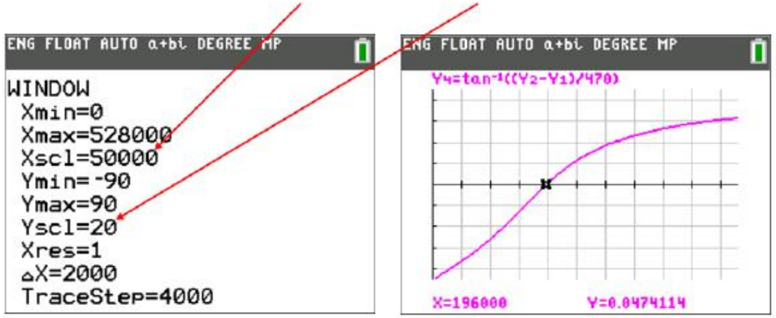
Summary: Series resonance (4 of 5)

The TI-84 Plus CE can find f_r by finding for the minimum point on the impedance curve. Press 2nd trace to enter the calculate menu. The result is shown here for the circuit shown earlier:



Summary: Phase angle

The phase angle is plotted on its own plot (because of the scale required to show it). Parameters for viewing the phase angle are shown. Note that Ymin is negative. The plot is from −90° to +90° with grid lines spacing at 50 kHz on *x* and 20° on *y*.



Summary: Series resonance (5 of 5)

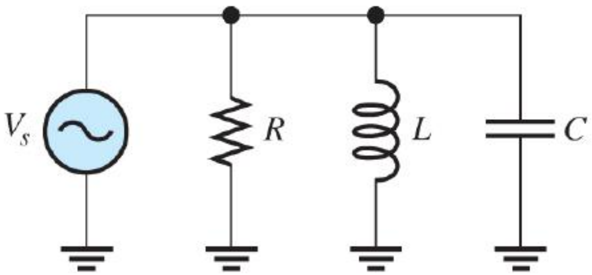
Summary of important concepts for series resonance:

- Capacitive and inductive reactances are equal.
- Total impedance is a minimum and is resistive.
- The current is maximum.
- The phase angle between V_S and I_S is zero.
- f_r is given by $f_r = \frac{1}{2\pi\sqrt{LC}}$

Summary: Impedance of parallel *RLC* circuits

For parallel *RLC* circuits, the impedance is found using the reciprocal of the sum-of-reciprocals.

$$\frac{1}{\mathbf{Z}} = \frac{1}{R\angle 0°} + \frac{1}{X_L\angle 90°} + \frac{1}{X_C\angle -90°}$$



Summary: Sinusoidal response of parallel *RLC* circuits (1 of 3)

For parallel *RLC* circuits, the current phasors can be obtained directly from Ohm's law. Recall that

$\mathbf{I}_R = \frac{\mathbf{V}_S}{\mathbf{R}}$ and that I_R is plotted along the positive real axis.

$\mathbf{I}_C = \frac{\mathbf{V}_S}{\mathbf{X}_C}$ and that I_C is plotted along the positive *j* axis.

$\mathbf{I}_L = \frac{\mathbf{V}_S}{\mathbf{X}_L}$ and that I_L is plotted along the negative *j* axis.

Summary: Sinusoidal response of parallel *RLC* circuits (2 of 3)

A typical current phasor diagram for a parallel *RLC* circuit is

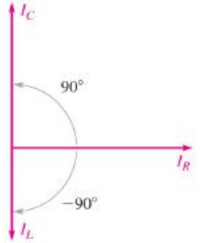
The total current is given by:

$$\mathbf{I}_{tot} = \sqrt{I_R^2 + (I_C - I_L)^2} \angle \tan^{-1}\left(\frac{I_{CL}}{I_R}\right)$$

Example:

What is $\mathbf{I}_{tot}$ if I_R = 10 mA, I_C = 15 mA and I_L = 5 mA?

$14.1\angle 45°$ mA



Pearson

Summary: Sinusoidal response of parallel *RLC* circuits (3 of 3)

The conductance, susceptance, and admittance phasors can also be used to find current. Recall that

$$\mathbf{G} = \frac{1}{\mathbf{R}} = \frac{1}{R\angle 0°}$$

$$\mathbf{B}_C = \frac{1}{X_C\angle -90°}$$

$$\mathbf{B}_L = \frac{1}{X_L\angle 90°}$$

$$\mathbf{Y} = \frac{1}{Z\angle \pm\theta}$$

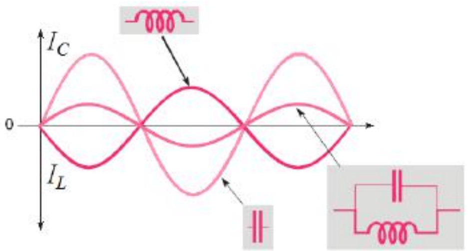
These quantities can be multiplied by the voltage to obtain current.

Pearson

Summary: Currents in a parallel *RLC* circuits (1 of 2)

The currents in the *RLC* components must add to the source current in accordance with KCL. Because of the opposite phase shift due to *L* and *C*, I_L and I_C effectively subtract.

Notice that I_C is out of phase with I_L. When they are algebraically added, the result is….



Pearson

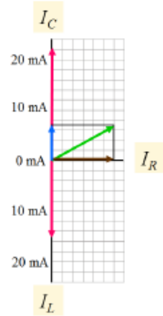
Summary: Currents in a parallel *RLC* circuits (2 of 2)

Example

Draw a diagram of the phasors if I_R = 12 mA, I_C = 22 mA and I_L = 15 mA.

Solution

- Set up a grid with a scale that will allow all of the data– say 2 mA/div.
- Plot the currents on the appropriate axes
- Combine the reactive currents
- Use the total reactive current and I_R to find the total current. In this case, $I_{tot} = 13.9\angle 30.3°$ mA

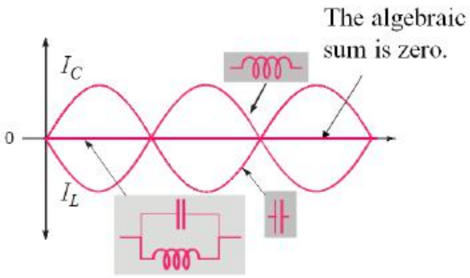


Pearson

Summary: Parallel resonance (1 of 4)

Ideally, at parallel resonance, I_C and I_L cancel because the currents are equal and opposite. The circuit is purely resistive at resonance.

Notice that I_C is out of phase with I_L. When they are algebraically added, the result is….



Pearson

Summary: Parallel resonance (2 of 4)

The formula for the resonant frequency in both parallel and series circuits is the same, namely

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$ (ideal case)

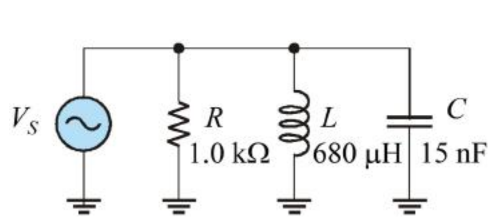
Example

What is the resonant frequency for the circuit?

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{(680\quad)(\quad)}}$$

$$= 49.8\ \text{kHz}$$



Pearson

Summary: Parallel resonance (3 of 4)

In practical circuits, when the coil resistance is considered, there is a small current at resonance and the resonant frequency is not exactly given by the ideal equation. The *Q* of the coil affects the equation for resonance:

$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{Q^2}{Q^2+1}}$$ (non-ideal)

For *Q* >10, the correction is less than 1%, and generally can be ignored.

Pearson

Summary: Parallel resonance (4 of 4)

Summary of important concepts for parallel resonance:

- Capacitive and inductive susceptance are equal.
- Total impedance is a maximum (ideally infinite).
- The current is minimum.
- The phase angle between V_S and I_S is zero.
- f_r is given by $f_r = \frac{1}{2\pi\sqrt{LC}}$

Pearson

Summary: Series-parallel *RLC* circuits (1 of 2)

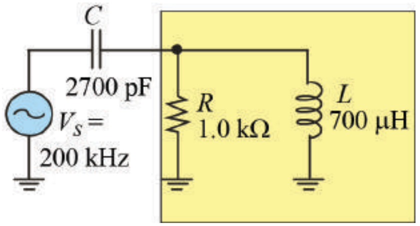
When *RLC* components are in series-parallel combinations, the rules from resistive circuits (but using complex numbers) apply.

For example, to find the total impedance of the circuit shown, you first calculate the impedance of the parallel combination (in the yellow box). Then add the result to the capacitor's reactance.

Example

Calculate the total impedance of the circuit.

See next slide….



Summary: Series-parallel *RLC* circuits (2 of 2)

Solution

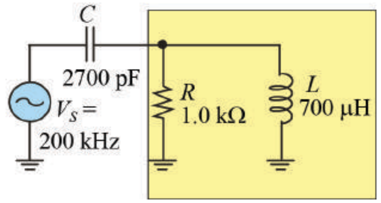
$$X_L = 2\pi fL = j0.880\ \text{k}\Omega \qquad X_C = \frac{1}{2\pi fC} = -j0.295\ \text{k}\Omega$$

The impedance of the yellow box is $Z_{ybox} = \dfrac{RX_L}{R + jX_L}$

$$Z_{ybox} = \left[\frac{(1.0\angle 0^\circ)(0.88\angle 90^\circ)}{1.0\ + j0.88}\right]\text{k}\Omega = \ 0.661\angle 48.7^\circ\ \text{k}\Omega = 0.436\ \text{k}\Omega + j0.496\ \text{k}\Omega$$

The total impedance is $Z_{tot} = X_C + Z_{ybox}$

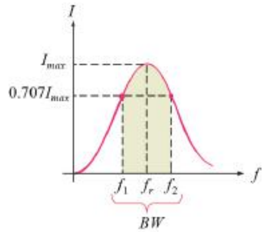
$$Z_{tot} = \left[-j0.295 + (0.436 + j0.496)\right]\text{k}\Omega$$
$$= \ 0.436\ \text{k}\Omega + j0.201\ \text{k}\Omega$$
$$= 0.480\angle 24.8^\circ\ \text{k}$$



Summary: Bandwidth of resonant circuits (1 of 2)

At the *series* resonant frequency, current is maximum. Bandwidth (*BW*) is the range of frequencies for which the current is equal to or greater than 70.7% of the maximum value.

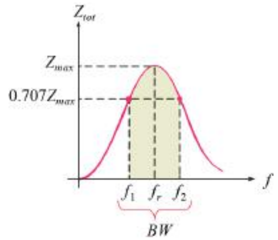
f_1 and f_2 are commonly referred to as the *critical frequencies, cutoff frequencies* or *half-power frequencies.*



Summary: Bandwidth of resonant circuits (2 of 2)

At the *parallel* resonant frequency, impedance is maximum, so current is a minimum at resonance. The bandwidth (*BW*) can be defined in terms of the impedance curve.

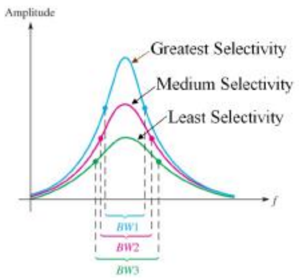
A parallel resonant circuit is commonly referred to as a *tank circuit* because of its ability to store energy like a storage tank.



Summary: Selectivity

Selectivity defines how well a resonant circuit responds to certain frequencies. The bandwidth is inversely proportional to *Q* in accordance with the formula,

$$BW = \frac{f_r}{Q}$$



Question

Which curve represents the highest *Q*?

The one with the greatest selectivity.

Key Terms (1 of 2)

Series resonance A condition in a series *RLC* circuit in which the reactances ideally cancel and the impedance is a minimum.

Resonant frequency (f_r) The frequency at which resonance occurs; also known as the *center frequency*.

Parallel resonance A condition in a parallel *RLC* circuit in which the reactances ideally are equal and the impedance is a maximum.

Tank circuit A parallel resonant circuit.

Key Terms (2 of 2)

Half-power frequency The frequency at which the output power of a resonant circuit is 50% of the maximum value (the output voltage is 70.7% of maximum); another name for critical or cutoff frequency.

Selectivity A measure of how effectively a resonant circuit passes desired frequencies and rejects all others. Generally, the narrower the bandwidth, the greater the selectivity.

Quiz (1 of 11)

1. In practical series and parallel resonant circuits, the total impedance of the circuit at resonance will be

   a. capacitive

   b. inductive

   c. resistive

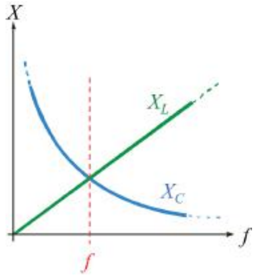
   d. none of the above

Quiz (2 of 11)

2. In a series resonant circuit, the current at the half-power frequency is
   a. maximum
   b. minimum
   c. 70.7% of the maximum value
   d. 70.7% of the minimum value

Quiz (3 of 11)

3. The frequency represented by the red dashed line is the
   a. resonant frequency
   b. half-power frequency
   c. critical frequency
   d. all of the above



Quiz (4 of 11)

4. In a series *RLC* circuit, if the frequency is below the resonant frequency, the circuit will appear to be
   a. capacitive
   b. inductive
   c. resistive
   d. answer depends on the particular components

Quiz (5 of 11)

5. In a series resonant circuit, the resonant frequency can be found from the equation
   a. $f_r = \frac{BW}{Q}$
   b. $f_r = \frac{1}{2\pi\sqrt{LC}}$
   c. $f_r = 0.707 I_{max}$
   d. $f_r = \frac{1}{2\pi LC}$

Quiz (6 of 11)

6. In an *ideal* parallel resonant circuit, the total impedance at resonance is

   a. zero
   b. equal to the resistance
   c. equal to the reactance
   d. infinite

Quiz (7 of 11)

7. In a parallel *RLC* circuit, the magnitude of the total current is always the

   a. same as the current in the resistor.
   b. phasor sum of all of the branch currents.
   c. same as the source current.
   d. difference between resistive and reactive currents.

Quiz (8 of 11)

8. If you increase the frequency in a parallel *RLC* circuit, the total current

   a. will not change
   b. will increase
   c. will decrease
   d. can increase or decrease depending on if it is above or below resonance.

Quiz (9 of 11)

9. The phase angle between the source voltage and current in a parallel *RLC* circuit will be positive if

   a. I_L is larger than I_C
   b. I_L is larger than I_R
   c. both a and b
   d. none of the above

Quiz (10 of 11)

10. A highly selectivity circuit will have a
    a. small *BW* and high *Q*.
    b. large *BW* and low *Q*.
    c. large *BW* and high *Q*.
    d. none of the above

Quiz (11 of 11)

Answers:

1. c
2. c
3. a
4. a
5. b
6. d
7. b
8. d
9. d
10. a