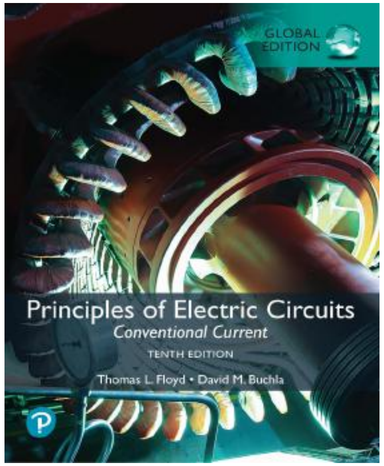
Principles of Electric Circuits: Conventional Current

Tenth Edition, Global Edition

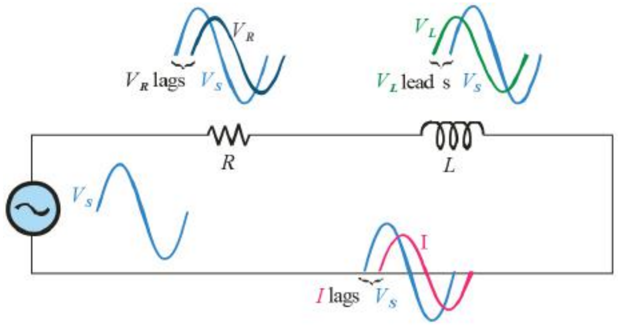


Chapter 15

RL Circuits

Summary: Sinusoidal response of series *RL* circuits

When both resistance and inductance are in a series circuit, the phase angle between the applied voltage and total current is between 0° and 90°, depending on the values of resistance and reactance.



Summary: Impedance of series *RL* circuits (1 of 2)

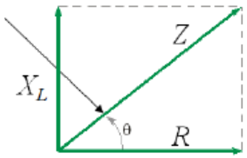
In a series *RL* circuit, the total impedance is the phasor sum of *R* and jX_L.

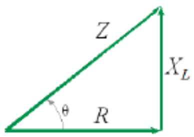
R is plotted along the positive x-axis.

X_L is plotted along the positive y-axis (+*j*).

$$\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Z is the diagonal





It is convenient to reposition the phasors into the *impedance triangle*.

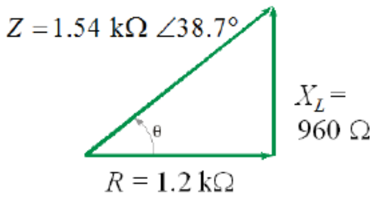
Summary: Impedance of series *RL* circuits (2 of 2)

Example

Sketch the impedance triangle and show the values for *R* = 1.2 kΩ and X_L = 960 Ω.

$$Z = \sqrt{(1.2\ \text{k}\Omega)^2 + (0.96\ \text{k}\Omega)^2}$$
$$= 1.54\ \text{k}\Omega$$

$$\theta = \tan^{-1}\frac{0.96\ \text{k}\Omega}{1.2\ \text{k}\Omega}$$
$$= +38.7°$$



Summary: Analysis of series *RL* circuits (1 of 2)

Ohm's law is applied to series *RL* circuits using phasor quantities of **Z**, **V**, and **I**.

$$\mathbf{V} = \mathbf{IZ} \qquad \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} \qquad \mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}}$$

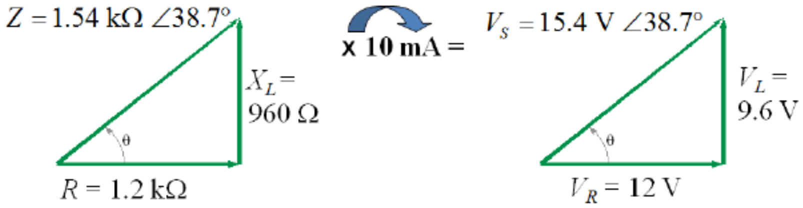
Because *I* is the same everywhere in a series circuit, you can obtain the voltage phasors by simply multiplying the impedance phasors by the current.

Summary: Analysis of series *RL* circuits (2 of 2)

Example

Assume the current in the previous example is 10 mA_{rms}. Sketch the voltage phasors. The impedance triangle from the previous example is shown for reference.

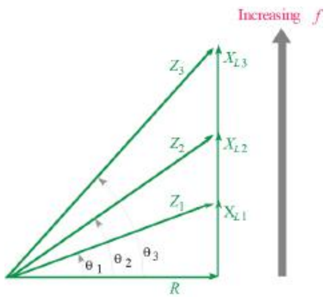
The voltage phasors can be found from Ohm's law. Multiply each impedance phasor by 10 mA.



Summary: Variation of phase angle with frequency

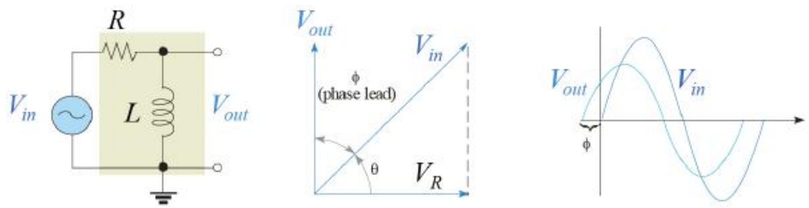
Phasor diagrams that have reactance phasors can be drawn for only a single frequency because X_L is a function of frequency.

As frequency changes, the impedance triangle for an *RL* circuit changes as illustrated here because X_L increases with increasing *f*. This determines the *frequency response* of *RL* circuits.



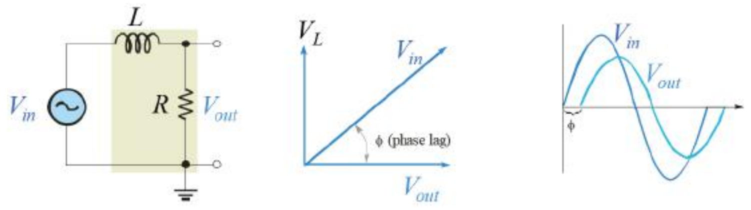
Summary: Applications (1 of 3)

For a given frequency, a series *RL* circuit can be used to produce a phase lead by a specific amount between an input voltage and an output by taking the output across the inductor. This circuit is also a basic high-pass filter, a circuit that passes high frequencies and rejects all others.



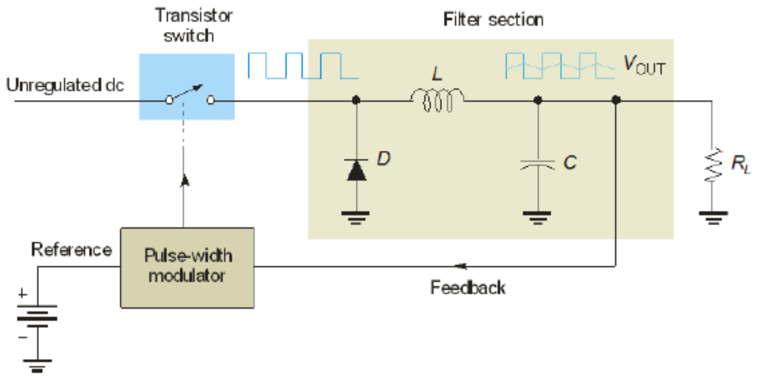
Summary: Applications (2 of 3)

Reversing the components in the previous circuit produces a circuit that is a basic lag network. This circuit is also a basic low-pass filter, a circuit that passes low frequencies and rejects all others.



Summary: Applications (3 of 3)

An common application for inductors is in switching regulators. The inductor serves as part of a filter network that opposes a *change* in current, as given by Lenz's law.



Summary: Sinusoidal response of parallel *RL* circuits (1 of 4)

For parallel circuits, it is useful to review conductance, susceptance, and admittance, introduced in Chapter 13.

Conductance is the reciprocal of resistance. $\mathbf{G} = \frac{1}{\mathbf{R}} = \frac{1}{R\angle 0°}$

Inductive susceptance is the reciprocal of inductive reactance. $\mathbf{B}_L = \frac{1}{\mathbf{X}_L}$

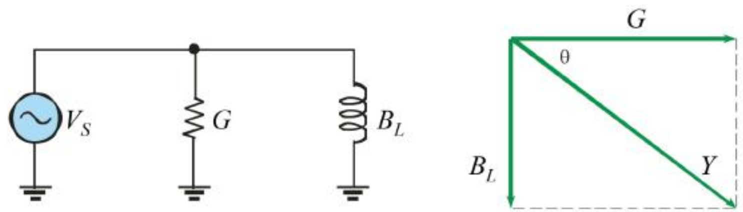
Admittance is the reciprocal of impedance. $\mathbf{Y} = \frac{1}{\mathbf{Z}}$

Summary: Sinusoidal response of parallel *RL* circuits (2 of 4)

In a parallel *RL* circuit, the admittance phasor is the sum of the conductance and inductive susceptance phasors. The magnitude can be expressed as

$$Y = \sqrt{G^2 + B_L^2}$$

From the diagram, the phase angle is $\theta = -\tan^{-1}\left(\frac{B_L}{G}\right)$

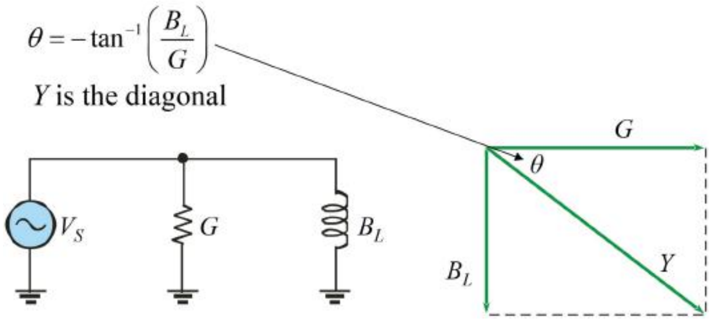


Summary: Sinusoidal response of parallel *RL* circuits (3 of 4)

Some important points to notice are:

G is plotted along the positive x-axis.

B_L is plotted along the negative y-axis (−j).



Summary: Sinusoidal response of parallel *RL* circuits (4 of 4)

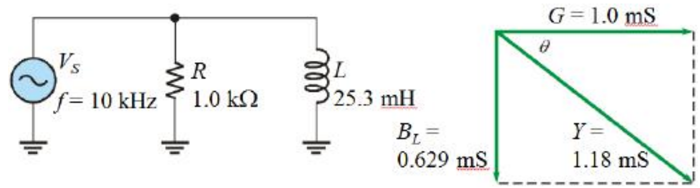
Example

Draw the admittance phasor diagram for the circuit.

The magnitude of the conductance and susceptance are:

$$G = \frac{1}{R} = \frac{1}{1.0\text{ k}\Omega} = 1.0\text{ mS} \quad B_L = \frac{1}{2\pi\,(10\text{ kHz})(25.3\text{ mH})} = 0.629\text{ mS}$$

$$Y = \sqrt{G^2 + B_L^{\,2}} = \sqrt{(1.0\text{ mS})^2 + (0.629\text{ mS})^2} = 1.18\text{ mS}$$



Summary: Analysis of series *RL* circuits

Ohm's law is applied to parallel *RL* circuits using phasor quantities of **Y**, **V**, and **I**.

$$\mathbf{Y} = \frac{\mathbf{I}}{\mathbf{V}} \quad \mathbf{V} = \frac{\mathbf{I}}{\mathbf{Y}} \quad \mathbf{I} = \mathbf{VY}$$

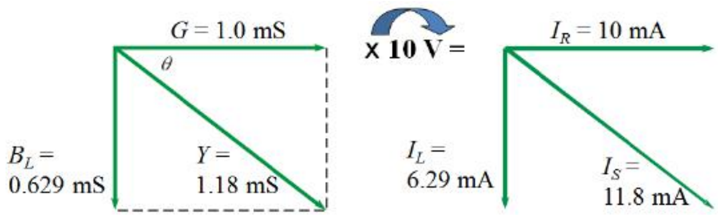
Because *V* is the same across all components in a parallel circuit, you can obtain the current phasors by simply multiplying the admittance phasors by the voltage.

Summary: Analysis of parallel RL circuits

Example

Assume the voltage in the previous example is 10 V. Sketch the current phasors. The admittance diagram from the previous example is shown for reference.

The current phasors can be found from Ohm's law. Multiply each admittance phasor by 10 V.

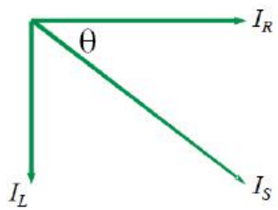


Summary: Phase angle of parallel *RL* circuits

Notice that the formula for inductive susceptance is the reciprocal of inductive reactance. Thus B_L and I_L are inversely proportional to *f*:

$$B_L = \frac{1}{2\pi fL}$$

As frequency increases, B_L and I_L decrease, so the angle between I_R and I_S must decrease as well.



Summary: Series-Parallel *RL* circuits

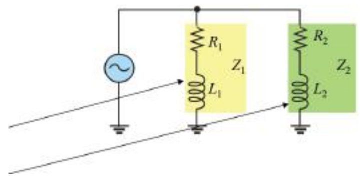
Series-parallel *RL* circuits are combinations of both series and parallel elements. The solution of these circuits is similar to resistive combinational circuits except complex numbers must be employed.

For example, the components in the yellow box and those in the green box are in series:

$$Z_1 = R_1 + X_{L1}$$

and

$$Z_2 = R_2 + X_{L2}$$

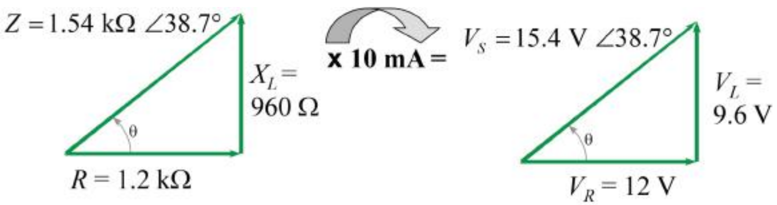


The two boxes are in parallel. Using phasor math, you can find the total impedance:

$$\mathbf{Z}_T = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1 + \mathbf{Z}_2}$$

Summary: The Power triangle (1 of 2)

Recall that in a series *RC* or *RL* circuit, you could multiply the impedance phasors by the current to obtain the voltage phasors. The earlier example from this chapter is shown for review:

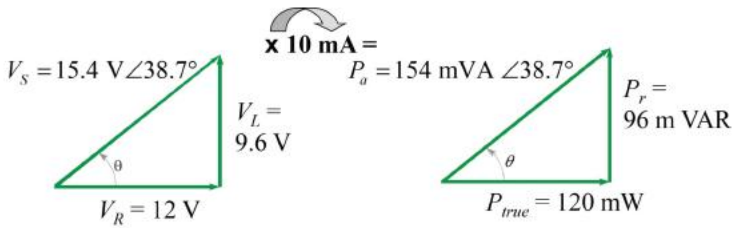


Summary: The Power triangle (2 of 2)

Multiplying the voltage phasors by I_{rms} gives the power triangle (equivalent to multiplying the impedance phasors by I^2). Apparent power is the product of the magnitude of the current and magnitude of the voltage and is plotted along the hypotenuse of the power triangle.

Example

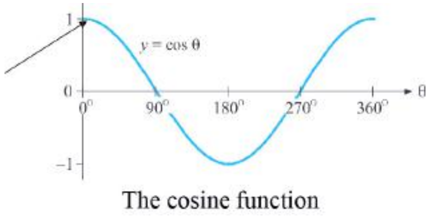
The rms current in the earlier example was 10 mA. Show the power triangle.



Summary: Power factor

The power factor applies to *RL* circuits as well as *RC* circuits. Recall that power factor was defined as *PF*= cos θ. To avoid wasting power, industrial plants try to maintain a *PF* near 1. This implies keeping the phase shift close to 0°, to maximize *PF*.

A graph of the mathematical cosine function is useful to observe. Notice when the phase angle is small the affect on the power factor is minimal as the function tends to be flatter for small angles.



The cosine function

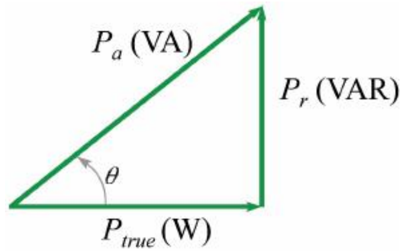
Question:

What is the *PF* when the phase angle is 18°? 0.95

Summary: Apparent power

Apparent power consists of two components; a true power component, that does the work, and a reactive power component, that is simply power shuttled back and forth between source and load.

Power factor corrections for an inductive load (motors, generators, etc.) are done by adding a parallel capacitor, which has a canceling effect.



Key Terms

Inductive susceptance (B_L) The ability of an inductor to permit current; the reciprocal of inductive reactance. The unit is the siemens (S).

RL lag circuit A phase shift circuit in which the output voltage, taken across the resistor, lags the input voltage by a specified angle.

RL lead circuit A phase shift circuit in which the output voltage, taken across the inductor, leads the input voltage by a specified angle.

Quiz (1 of 11)

1. In a series *RL* circuit, the resistance phasor is plotted along the

   a. positive real axis
   b. negative real axis
   c. positive imaginary axis
   d. negative imaginary axis

Quiz (2 of 11)

2. In a series *RL* circuit, there is a frequency at which the magnitude of the inductive reactance is equal to the resistance. At this frequency, the phase shift between the source voltage and source current is

   a. 0°
   b. 45°
   c. 90°
   d. 120°

Quiz (3 of 11)

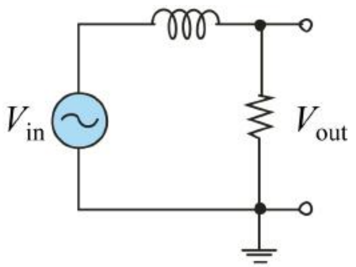
3. If you multiply each of the impedance phasors in a series *RL* circuit by the current, the result is the

   a. voltage phasors
   b. power phasors
   c. admittance phasors
   d. none of the above

Quiz (4 of 11)

4. The circuit shown is

   a. a lead network
   b. a low-pass filter
   c. both of the above
   d. none of the above



Quiz (5 of 11)

5. In a series *RL* circuit, the phase angle can be found from the equation

   a. $\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$
   b. $\theta = \tan^{-1}\left(\frac{V_L}{V_R}\right)$
   c. both of the above are correct
   d. none of the above is correct

Quiz (6 of 11)

6. In an *RL* parallel circuit, the inductive susceptance is plotted on the –*j* axis. The reason for this is that

   a. the phase angle between V_S and I is –90°
   b. inductors are not ideal components
   c. current leads voltage in an inductor
   d. all of the above

Quiz (7 of 11)

7. In a parallel *RL* circuit, the magnitude of the admittance phasor can be expressed as

   a. $Y = \dfrac{1}{\dfrac{1}{G} + \dfrac{1}{B_L}}$
   b. $Y = \sqrt{G^2 - B_L^2}$
   c. $Y = G + B_L$
   d. $Y = \sqrt{G^2 + B_L^2}$

Quiz (8 of 11)

8. If you increase the frequency in a parallel *RL* circuit,

   a. the total admittance will increase
   b. the total current will increase
   c. both a and b
   d. none of the above

Quiz (9 of 11)

9. The phase angle between the source voltage and current in a parallel *RL* circuit will increase if

   a. the resistance is larger
   b. the inductance is larger
   c. both a and b
   d. none of the above

Quiz (10 of 11)

10. A power factor of zero implies that the
    a. circuit is entirely reactive
    b. reactive and true power are equal
    c. circuit is entirely resistive
    d. maximum power is delivered to the load

Quiz (11 of 11)

Answers:

1. a
2. b
3. a
4. b
5. c
6. a
7. d
8. d
9. a
10. a