AI-C 205 Probability and Statistics

Homework 3
(Deadline: 20^{th} Oct, 2026)

1. What is the expected value, variance, and mode of X with the following probability distribution?

x	0	1	2
$P(x)$	0.5	0	0.5

2. Let us say we want to predict earthquakes. Assume we have a system connected to a number of sensors returning every day a probability estimate, p, of an earthquake the following day. Depending on p, we need to decide on whether to issue an alert to the population or not. We do not want to generate unnecessary alerts so as not to panic the population unnecessarily, and we certainly do not want to miss any earthquakes, which is a much worse outcome. Let us say that if the cost of an unnecessary alert is 1, the cost of an unannounced earthquake is c, for what values of p should we issue an alert?

3. Show that
$$E[X^2] \geq E[X]^2$$

4. Show that Corr(X, Y) is between –1 and +1, and also that if Corr(X, Y) = ± 1, we can write $Y = a + bX$.

5. Prove Chebyshev's inequality using Markov's inequality.